

Research Article

CRITICAL TEMPERATURE AND CONDENSATE FRACTION OF A BOSE – EINSTEIN CONDENSATE TRAPPED IN A FINITE VOLUME

Mukoya AK¹, *Sakwa TW^{2,3}, Khanna KM⁴, Ayodo YK², Sarai A², Rapando BW² and Namwitako J⁴

¹*Department of Physics University of Karatina, P.O Box 1957-10101, Karatina, Kenya*

²*Department of Physics Masinde Muliro University of Science and Technology, P.O Box 190-50100, Kakamega, Kenya*

³*Department of Physical Sciences University of Kabianga, P.O Box 2030-20200, Kericho, Kenya*

⁴*Department of Physics, University of Eldoret, P.O Box 1125, Eldoret, Kenya*

**Author for Correspondence*

ABSTRACT

Bose - Einstein Condensation, BEC, of an ideal gas is investigated for a finite number of particles trapped in a harmonic potential. The values of the critical temperature for Rb-87, its condensate fraction and the energy per particle are determined. The maxima appear to increase with the number of particles due to the fact that a smaller system has a larger available effective volume which concurs with the behavior of the critical temperatures.

Keywords: *Bose Einstein Condensation, Condensate Fraction, Critical Temperature*

INTRODUCTION

The phenomenon of Bose-Einstein Condensation (BEC) has a rich history over the past century, beginning with the recognition of two classes of particles in nature, bosons and fermions, obeying Bose-Einstein and Fermi-Dirac statistics, respectively.

The different particle statistics have fundamental and far-reaching physical consequences. However, effects become evident only at low temperatures where particles gather in the lowest energy state of a system and the average occupation of a state approaches unity for fermions, but for bosons the lowest state may contain a large fraction of particles.

A BEC is a state of matter formed by a system of bosons confined in an external potential and cooled to a temperature very near to Zero Kelvin temperature. BEC is a purely quantum-statistical phase transition characterized by a macroscopic population of ground state below the transition temperature. This state is called the zero-momentum-state (ZMS).

The existence of BEC was proven experimentally in helium and other elements from measurements of momentum distribution (Grossmann and Holthaus, 1995) and in semiconductors, where para-excitons were found to condense. Pure BEC was further observed in systems very different from Bagnato *et al.*, (1969) He, namely dilute alkali gases.

The experimental realization of BEC in alkali-metal atoms and thereafter in atomic hydrogen stimulated new interest in the theoretical study of inhomogeneous Bose gas properties (Killian *et al.*, 1998). Thermodynamic properties have been investigated by several authors for the trapped Bose gas (Dalfovo *et al.*, 1999).

The demonstration of BEC in gases of Rubidium (Anderson *et al.*, 1995; Myact *et al.*, 1997; Ernest *et al.*, 1998), Lithium (Bradley *et al.*, 1995), and then Sodium (Davis *et al.*, 1995) was impressive achievement that stimulated a lot of theoretical studies. For many purposes the complicated magnetic traps used in the experiments is approximated by harmonic oscillator potentials. There are many studies of BEC in harmonic oscillator confining potentials (Grossmann and Holthaus, 1995; Bagnato *et al.*, 1991; Haugerud *et al.*, 1997).

From the theoretical point of view, due to the effects of confining potential traps these atomic systems can be modeled by harmonic oscillator potential. However, the behavior of the 3D harmonic trap is the most relevant aspect for experiments.

Research Article

Theoretical studies show that realization of the quantum phase transition can be more robust with a quartic potential (Fetter, 2001) with the condensation temperature and the thermodynamic properties of a rotating ideal Bose gas in an anharmonic trap being studied (Kling and Pelster, 2007) appreciably. However, in the experimental realization of the quadratic plus quartic confining potential, the observation of the fast rotating vortices eludes an unambiguous detection (Bretin *et al.*, 2004).

BECs of trapped alkali-metal atoms were achieved at temperatures as low as 100 nanokelvin. The number of particles in the ground state of the traps ranged from a few thousand (Anderson *et al.*, 1995) to a few millions (Davis *et al.*, 1995).

For a given number of Bosons, the temperature of the system is reduced below a certain critical temperature leading to an accumulation of particles in the ground state energy thus increasing the condensate fraction (Bretin *et al.*, 2004). This implies that the number of particles in the condensate varies considerably with temperature especially in the ultra cold temperature regime. Some of the implications of a finite number of particles to the condensate fraction and the critical temperature can, therefore, be studied.

Critical Temperature and Condensate Fraction of BEC

For an assembly described by Bose-Einstein statistics, the probability that a given energy will be occupied is specified by the Bose-Einstein distribution. When the temperature is gradually reduced, the ground state will quite suddenly become occupied by a significant fraction of all the particles in the assembly. From the conventional total number of particles in the assembly, the levels which are closely spaced in energy, necessitates the use of appropriate density of states function which in our calculations shows that

$N_e \leq \frac{41.1V}{h^3} (mk_B T)^{\frac{3}{2}}$. As the temperature is reduced, a situation is likely to occur at some critical

temperature T_c when the value of N_e reduces to a value smaller than the total number of particles in the assembly. Hence for $T \leq T_c$, the rest of the particles settle in the zero-momentum ground state.

For $T \leq T_c$, the chemical potential diminishes, implying that N_e stays as close as possible to its maximum value. This condition was observed in this work to give a definition of the T_c value from which the reduced temperature was determined and fitted in the conventional expression of the total number of

particles in an ensemble to give T_c value as $T_c = \frac{h^2}{11.9mk_B} \left(\frac{N}{V}\right)^{\frac{2}{3}}$, where $\frac{N}{V}$ is the number of particles per unit volume.

We build on the restriction of the three level approximation as developed by Sakwa *et al.*, (2004) and gave a simple physical meaning that the second and third terms denotes the maximum number of particles which can be accommodated in excited states when the fugacity approaches one. All particles exceeding this maximum number must “condense” to the ground state from which the critical temperature T_c and the condensate fraction were determined.

A phase transition occurs in the thermodynamic limit and is usually defined by singularities and critical behavior.

For the finite-N system, we adopt the treatment described by Bagnato *et al.*, (1969) and take the macroscopic occupation of the ground state as the defining characteristic of BEC phase transition.

This calculations therefore assumes that particles “condense” into the ground state of the trap when the temperature becomes sufficiently low i.e. as $T \rightarrow 0$, then $N \rightarrow N_0$. Starting with $T > T_c$, then as $T \rightarrow T_c$ we have at the onset of condensation, $N_0 \approx 0$ i.e. at critical temperature the chemical potential vanishes as indicated by Khanna *et al.*, (1967).

In the case of bosons, the number of particles in the excited states, N_e , is equal to the total number of particles, N , constituting the system when $T > T_c$. Finally, for a large number of particles, the condensate fraction of a trapped boson system was determined to a good approximation as $\frac{N_0}{N} = 1 - \left(\frac{T}{T_0}\right)^3$. The

Research Article

critical temperature for bosons trapped in a finite volume is now arrived at on using Cardano’s formula in a rather rigorous calculation.

Using the work of (Grossmann and Holthaus, 1995) we computed the expression for mean energy U of N non interacting bosons that are harmonically trapped. By using the density of state formula the release energy is determined directly from the root mean square cloud radius and the time-of-flight. The non-interacting model predicts a released energy independent of N. Conversely, the observed release energy per particle depends strongly on N. For temperatures below the BEC transition temperature, the chemical potential has a contribution to the specific heat capacity. At $T < T_c$, and for large value of N, the specific heat capacity can be expressed in terms of the reduced temperature $\frac{C(T)}{Nk_B} = 10.81 \left(\frac{T}{T_0}\right)^3 + \frac{9.569}{N^{\frac{1}{3}}} \left(\frac{T}{T_0}\right)^2$.

RESULTS AND DISCUSSION

The condensate fraction of a Bose condensate varies with the reduced temperature for a given value of the number of particles in the condensate down to absolute zero temperature as shown in Figure 1. As $T \rightarrow 0$ the exponential suppression of excited particles becomes stronger and stronger diminishing particle in the excited states. At $T < \frac{\hbar\omega}{k_B}$, there will be hardly any particles in the excited states; most of the particles will be in the lowest energy state. This shows that as $T \rightarrow 0, N_o \rightarrow N$.

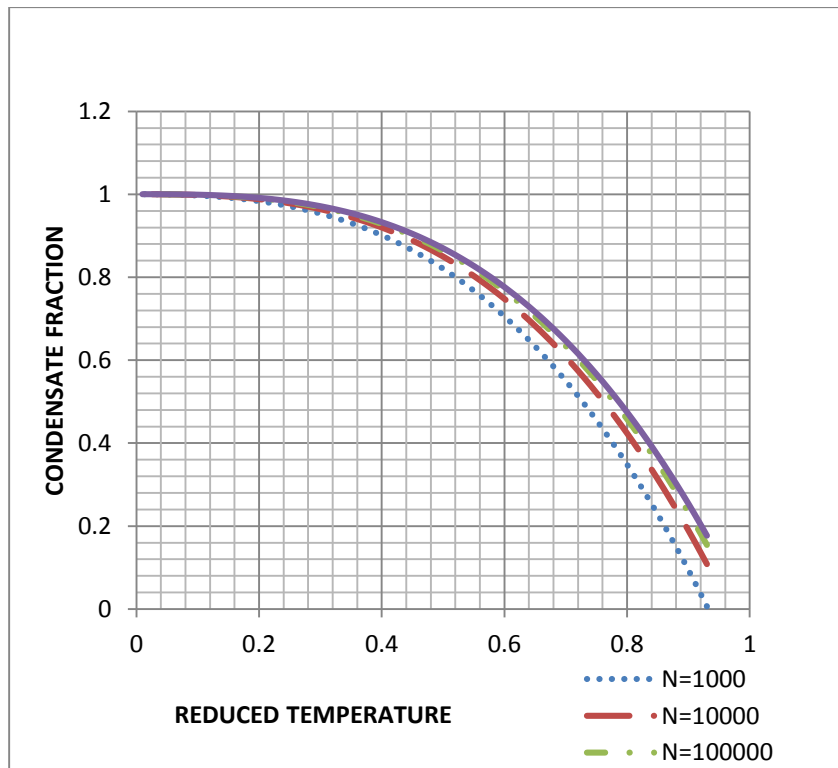


Figure 1: Graphs of Condensate Fraction against Reduced Temperature for $N=10^3$, $N=10^4$, $N=10^5$ and $N=10^6$.

The specific heat capacity is found to increase asymptotically as the reduced temperature approaches unity (See Figure 2). A maximum value corresponds to the temperature at which BEC occurs and is the transition temperature T_c . At temperature greater than T_c the specific heat capacity falls rapidly as shown in Figure 3.

Research Article

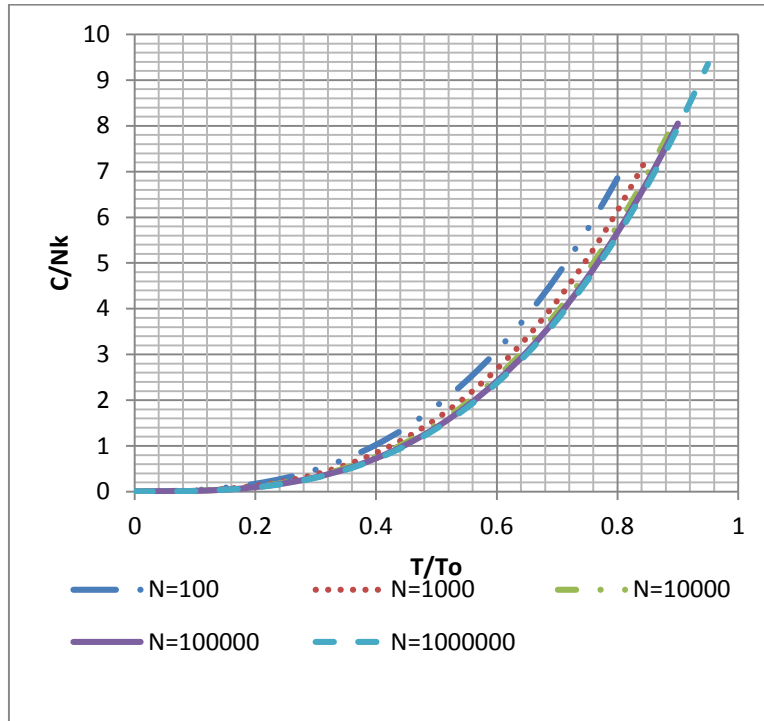


Figure 2: Graphs of specific heat capacity against reduced temperature for different values of N

The kink in Figure 3 occurs at a reduced temperature $\frac{T}{T_c} < 1$. As N decreases, the specific heat capacities corresponding to the maxima are at values of $\frac{T_c}{T_0}$. This implies that the transition temperature is influenced by the value of N. Also notably, the values of the maxima appear to decrease as N decreases in agreement with the behavior of the critical temperatures. This is due to the fact that a smaller system has a larger available effective volume (Bagnato *et al.*, 1987).

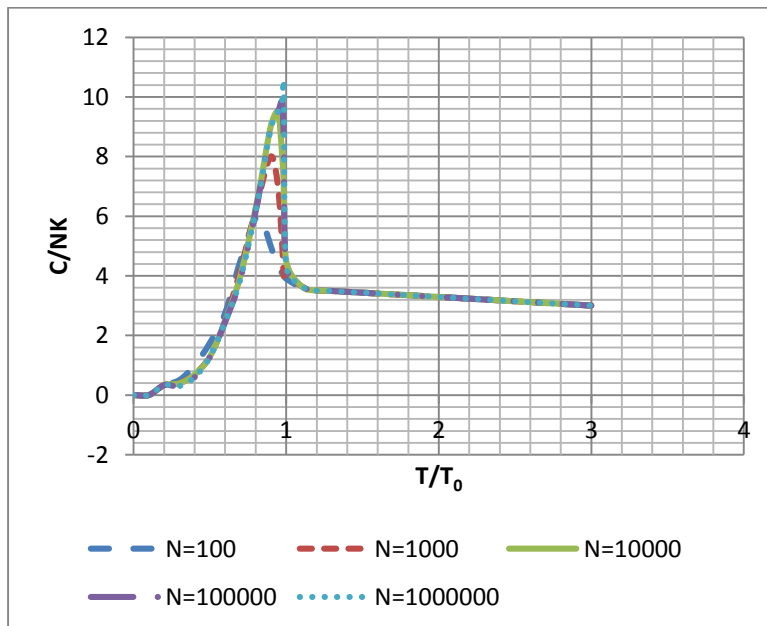


Figure 3: Graphs of $\frac{C}{Nk_B}$ against $\frac{T}{T_0}$ for different values of N

Research Article

Now using the oscillator frequencies given by Bradley *et al.*, (1995) for Lithium, and taking $N=2 \times 10^5$, heat capacity gives a maximum at $\frac{T}{T_0} = 0.983$. This translates to $T_c \approx 380nK$ which is in good agreement with the range of 100-400nK for the experiment. For Rubidium with $N=2 \times 10^4$, with the oscillator frequency given by Anderson *et al.*, (1995) (i.e. $\bar{\omega}=379.28$ Hz), the heat capacity has a maximum value at $\frac{T}{T_0} = 0.960$. This gives $T_c \approx 71 nK$. Using the frequency by Davis *et al.*, (1995) i.e. $\frac{\omega}{2\pi} = 416$ Hz and taking $N = 5 \times 10^5$, then maximum heat capacity occurs at $\frac{T}{T_0} = 0.985$ for Sodium and implying that $T_c \approx 1.47 \times 10^{-6} K$ which again agrees with the value of $2 \times 10^{-6} K$ by Davis *et al.*, (1995). The inclusion of interactions between the particles profoundly changes the nature of the BEC phase transition and is important for the occurrence of a macroscopic phase. Inter-particle interactions have an effect on the critical temperature and condensate fraction and may be studied further.

ACKNOWLEDGEMENT

We appreciate our Universities who have given us platforms to share our talents and skills as their dedicated employees.

REFERENCES

- Anderson MH, Ensher JR, Matthews MR, Wieman CE and Cornell EA (1995).** Observation of Bose-Einstein Condensation in a dilute atomic vapor. *Science* **269** 198.
- Bagnato V, Pritchard DE and Kleppner D (1987).** Bose-Einstein Condensation in an external potential. *Physical Review A* **35** 4354.
- Bagnato VS and Kleppner D (1991).** Bose-Einstein Condensation in low-dimensional traps. *Physical Review A* **44** 7439.
- Bagnato VS, Napolitano R and De Luca J (1969).** Effect of a finite number of particles in the Bose-Einstein Condensation of a trapped gas. *Physical Review A* **54** 656 (1996).
- Bradley CC, Sackett CA, Tollett JJ and Hulet RG (1995).** Evidence of Bose-Einstein Condensation in Gas with attractive interactions. *Physical Review Letters* **75** 1687.
- Bretin V, Stock S, Seurin Y and Dalibard J (2004).** Rotating Fermi gas in anharmonic trap. *Physical Review Letters* **92** 050403.
- Dalfovo F, Giorgini S, Pitaevskii LP and Stringari S (1999).** Theory of Bose-Einstein Condensation in trapped Gases. *Review of Modern Physics* **71** 463.
- Davis KB, Mewes MO, Andrews MR, Van Druten NJ, Durfee DS, Kurn DM and Ketterle W (1995).** Bose-Einstein Condensation in a Gas of Sodium Atoms. *Physical Review Letters* **75** 3969.
- Ernst U, Marte A, Schreck F, Schuster J and Rempe G (1998).** Bose-Einstein condensation in a pure Ioffe-Pritchard field configuration. *Europhysics Letters* **41**(1) 1-6.
- Fetter AL (2001).** Rotating vortex lattice in a Bose Einstein Condensate Trapped in combined quadratic and quartic radial potentials. *Physical Review A* **64** 063608.
- Groot SR, Hooyman GJ and Seldam CA (1950).** Density of states for Bose-Einstein condensation in harmonic oscillator potentials. *Proceedings of the Royal Society (London)* **A 203** 266.
- Grossmann S and Holthaus M (1995).** Bose-Einstein condensation in harmonic traps. *Physics Letters A* **208** 188.
- Haugerud H, Haugset T and Ravndal F (1997).** A more accurate analysis of Bose-Einstein Condensation in harmonic traps. *Physics Letters A* **225** 18-22.
- Khanna KM (1967).** Alpha-Particle Model for Nuclear Matter. *Journal of Physical Society of Japan* **23** 1429.
- Killian TC, Fried DG, Willmann L, Landhuis D, Moss SC, Greytak TJ and Kleppner D (1998).** Bose Einstein condensation of Atomic Hydrogen. *Physical Review Letters* **81** 3811.
- Kling S and Pelster A (2007).** Thermodynamical properties of a rotating ideal Bose gas. *Physical Review A* **76** 023609.

Research Article

Myact CJ, Burt EA, Ghrist RW, Cornell EA and Wieman CE (1997). Bose Einstein Condensation in dilute gases. *Physical Review Letters* **78** 586.

Sakwa WT, Khanna KM, Mueni M, Rotich S and Torogey PK (2004). Four Level Approximation in Disordered Medium. *Indian Journal of Pure and Applied Physics* **42** 355-360.