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Research Article

Mostar Index of Cycle-Related Structures

Fatima Asif,[1] **Agha Kashif**,[1] **Sohail Zafar**,[1] **and Michael Onyango Ojiema**[2]

[1]*Department of Mathematics, University of Management and Technology (UMT), Lahore, Pakistan*
[2]*Masinde Muliro University of Science and Technology, Kakamega, Kenya*

Correspondence should be addressed to Michael Onyango Ojiema; mojiema@mmust.ac.ke

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A topological index is a numerical quantity associated with the molecular structure of a chemical compound. This number remains fixed with respect to the symmetry of a molecular graph. Diverse research studies have shown that the topological indices of symmetrical graphs are interrelated with several physiochemical properties such as boiling point, density, and heat of formation. Peripherality is also an important tool to study topological aspects of molecular graphs. Recently, a bond-additive topological index called the Mostar index that measures the peripherality of a graph is investigated which attained wide attention of researchers. In this article, we compute the Mostar index of cycle-related structures such as the Jahangir graph and the cycle graph with chord.

1. Introduction and Preliminaries

Graph theory is being extensively used in mathematical chemistry for the numerical formulation of chemical compounds by representing atoms as vertices and bonds as edges. The topological index (TI) of a molecular graph is a numerical quantity associated with the molecular structure of a chemical compound [1, 2]. These quantities are well correlated with physicochemical properties and are used as a tool to predict quantitative structure-activity relationships and quantitative structure-property relationships (QSAR/QSPR) [3, 4]. QSAR and QSPR techniques have been widely used to study the structural properties of a molecule and its biological activity [5, 6]. The TIs are majorly classified into two types, namely, degree-based and distance-based or bond-additive topological indices. The degree-based TIs focus on the role of incident bonds towards the molecular structure [7, 8] whereas the distance-based TIs emphasize on the contribution of distances between atoms towards the structure of a compound [9]. The introduction of the first distance-based TI by Wiener in [10] pointed its significance towards the physicochemical properties of the compound. The physicochemical properties such as boiling point and melting point were shown in correlation with the Wiener index [11]. Afterwards, distance-based TIs such as the Hosaya, Shultz, and Szeged indexes were introduced [12, 13]. The bond-additive TIs have several applications; for example, they are useful in pharmaceutical sciences, in the prediction of physical properties of a molecule, and in complex network theory [14, 15]. To study further topological properties of various chemical compounds, we refer to [16, 17].

In this article, we will consider a simple and finite graph G with a vertex set $V(G)$ and an edge set $E(G)$. The degree of any vertex x, denoted by d_x, is the number of its incident vertices. The distance between any two vertices $x, y \in V(G)$, denoted by $d(x, y)$, is the length of a shortest path connecting them. For any edge $xy \in E(G)$, the collection of all vertices that are nearer to x than y will be called the resolving neighborhood of (x, y), denoted by $\eta(x, y)$, and its cardinality is denoted by η_{xy}.

Recently, to measure the peripherality of graphs, Doslic et al. proposed a new bond-additive topological index called the Mostar index [18]. It is defined as

$$Mo(G) = \sum_{xy\in E(G)} \left|n_x - n_y\right|. \tag{1}$$

The Mostar index measures peripheral atoms and bonds to determine the physical and chemical properties of a molecular graph. Each edge (bond) is peripheral if there are an unequal number of vertices (atoms) in each neighborhood of its end vertices. In chemistry, peripherality is used to measure the nonbalancedness among the bonds of a chemical graph. Recently, many researchers took the initiative to investigate the chemical properties and mathematical perspectives of the Mostar index. Arockiaraj et al. defined the edge Mostar index and computed the Mostar index and edge Mostar index of carbon nanostructures [19]. Hayat and Zhou determined the maximum Mostar index of all $n-$ vertex cacti and upper bound for the Mostar index of $n-$ vertex cacti that contains $k-$ cycles [20]. Huang et al. computed the Mostar index to find the extremal hexagonal chains [21]. The conjecture about the bicyclic graphs characterized by Doslic et al. has been proved by Tepeh [22].

The Mostar index of a graph G can be defined as

$$Mo(G) = \sum_{xy\in E(G)} Mo(xy), \tag{2}$$

where

$$Mo(xy) = \left|\eta_{xy} - \eta_{yx}\right|. \tag{3}$$

It is an NP-hard problem to derive general formulas for the Mostar index; therefore, Doslic et al. computed the Mostar index for different classes of graphs. Further, topological properties of some interesting symmetrical structures are computed in the articles [23, 24]. This motivates us to compute the Mostar index of cycle-related graphs such as the Jahangir graph and the cycle graph with chord.

1.1. Main Results. The main results computed in this article are as follows.

Theorem 1. *Consider the graphs $J_{n,m}$ and $C_{n,t}$; then,*

(1) For $n, m \geq 5$,

$$Mo\left(J_{n,m}\right) = \begin{cases} m^2n^2 + 2m^2n - mn^2 + m^2 - 6mn - 5m, & \text{if } n = \text{odd}, \\ m^2n^2 + 2m^2n - mn^2 + m^2 - 5mn - m, & \text{if } n = \text{even}. \end{cases} \tag{4}$$

(2) For $n \geq 9$ and $t \geq 3$,

$$Mo\left(C_{n,t}\right) = \begin{cases} 2(t-2)(n-t), & \text{if } n = \text{odd}, \\ 2(n-t)(t-2), & \text{if } n = \text{even},\ t = \text{odd}, \\ 2(n-t)(t-2) - 2, & \text{if } n = \text{even},\ t = \text{even}. \end{cases} \tag{5}$$

2. The Mostar Index of the Jahangir Graph

In this section, the Mostar index of the family of the Jahangir graph $J_{n,m}$ for $n, m \geq 5$ is computed. The Jahangir graph $J_{n,m}$ consists of a cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} at distance n to each other on C_{nm} as shown in Figure 1. The vertex set of $J_{n,m}$ is $V(J_{n,m}) = \{u, w_1, w_2, \ldots, w_m\} \cup \{v_1, v_2, \ldots, v_{nm}\}$. The edge set of $J_{n,m}$ is $E(J_{n,m}) = \{uw_k|1 \leq k \leq m\} \cup \ w_1v_1, w_1v_{nm}, w_kv_{(k-1)n}, w_k\ v_{(k-1)n+1}|2 \leq k \leq m\} \ \cup \left\{v_{ni+1}v_{ni+2}, \ldots, v_{ni+(n-1)}\right.$ $v_{ni+n}|0 \leq i \leq m-1\}$. The edge set of $J_{n,m}$ is partitioned as follows.

For n = even,

$$\begin{aligned} E_1\left(J_{n,m}\right) &= \left\{uw_k|1 \leq k \leq m\right\}, \\ E_2\left(J_{n,m}\right) &= \left\{w_kv_{(k-1)n}, w_kv_{(k-1)n+1}|2 \leq k \leq m\right\} \cup \left\{w_1v_1, w_1v_{nm}\right\}, \\ E_3\left(J_{n,m}\right) &= \left\{v_{ni+1}v_{ni+2}, \ldots, v_{((2i+1)n-4)/2}v_{((2i+1)n-2)/2}, v_{((2i+1)n+4)/2}v_{((2i+1)n+6)/2}-\right. \\ &\quad \left., \ldots, v_{ni+(n-1)}v_{ni+n}|0 \leq i \leq m-1\right\}, \\ E_4\left(J_{n,m}\right) &= \left\{v_{((2i+1)n-2)/2}v_{((2i+1)n)/2}, v_{((2i+1)n+2)/2}v_{((2i+1)n+4)/2}|0 \leq i \leq m-1\right\}, \\ E_5\left(J_{n,m}\right) &= \left\{v_{((2i+1)n)/2}v_{((2i+1)n+2)/2}|0 \leq i \leq m-1\right\}. \end{aligned} \tag{6}$$

For n = odd,

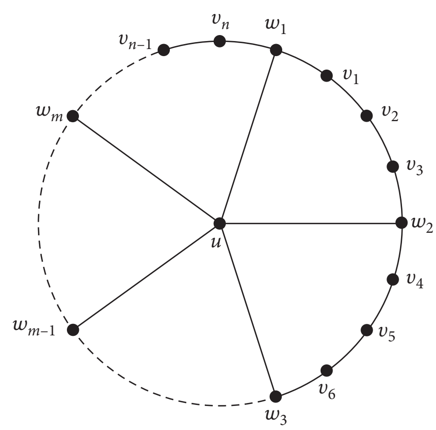

FIGURE 1: The Jahangir graph $J_{3,m}$.

$$\begin{aligned} E_1'(J_{n,m}) &= \{uw_k | 1 \le k \le m\}, \\ E_2'(J_{n,m}) &= \{w_k v_{(k-1)n}, w_k v_{(k-1)n+1} | 2 \le k \le m\} \cup \{w_1 v_1, w_1 v_{nm}\}, \\ E_3'(J_{n,m}) &= \{v_{ni+1} v_{ni+2}, \ldots, v_{((2i+1)n-3)/2} v_{((2i+1)n-1)/2}, v_{((2i+1)n+3)/2} v_{((2i+1)n+5)/2} - \\ &\quad , \ldots, v_{ni+(n-1)} v_{ni+n} | 0 \le i \le m-1\}, \\ E_4'(J_{n,m}) &= \{v_{((2i+1)n-1)/2} v_{((2i+1)n+1)/2}, v_{((2i+1)n+1)/2} v_{((2i+1)n+3)/2} | 0 \le i \le m-1\}. \end{aligned} \tag{7}$$

In the following lemmas, the Mostar index corresponding to equations (6) and (7) of $J_{n,m}$ is computed.

Lemma 1. *For $xy \in E(J_{n,m})$ where n is even and $n, m \ge 5$, then*

$$Mo(xy) = \begin{cases} mn - 2n + m - 3, & \text{if } xy \in E_1(J_{n,m}), \\ mn + m - n - 1, & \text{if } xy \in E_2(J_{n,m}), \\ mn - n + m - 2, & \text{if } xy \in E_3(J_{n,m}), \\ \dfrac{2mn + 2m - 3n - 4}{2}, & \text{if } xy \in E_4(J_{n,m}), \\ 0, & \text{if } xy \in E_5(J_{n,m}). \end{cases} \tag{8}$$

Proof. The symmetry of graph $J_{n,m}$ allows us to discuss the Mostar index of only the following types of edges:

Case (i) (for $xy \in E_1(J_{m,n})$): let $x = u$ and $y = w_1$, the resolving neighborhoods of (u, w_1) and (w_1, u) are $\{u, w_2, \ldots, w_m, v_{n+4/2}, \ldots, v_{mn-(n+2/2)}\}$ and $\{w_1, v_1, \ldots, v_{n/2}, v_{mn-(n-2/2)}-, \ldots, v_{mn}\}$, respectively. Hence, $\eta_{uw_1} = mn - n + m - 2$ and $\eta_{w_1 u} = n + 1$. Therefore, in view of equation (3), $Mo(uw_1) = mn - 2n + m - 3$.

Case (ii) (for $xy \in E_2(J_{m,n})$): let $x = v_1$ and $y = w_1$, by means of Figure 1, $\eta(v_1, w_1) = \{v_1, \ldots, v_{n+2/2}\}$ and $\eta(w_1, v_1) = \{u, w_1, \ldots, w_m, v_{n+5/2}, \ldots, v_{mn}\}$. Hence, $\eta_{v_1 w_1} = (n + 2/2)$ and $\eta_{w_1 v_1} = (2mn - n + 2m - 1)/2$. Therefore, using equation (3), $Mo(v_1 w_1) = mn + m - n - 1$.

Case (iii) (for $xy \in E_3(J_{m,n})$): let $x = v_i$ and $y = v_{i+1}$ for $1 \le i \le (n - 3/2)$, the resolving neighborhoods of (v_i, v_{i+1}) and (v_i, v_{i+1}) are $\{u, w_1, \ldots, w_m, v_1, \ldots, v_i, v_{(n+2i+6)/2}, \ldots, v_{mn}\}$ and $\{v_{i+1}, \ldots, v_{n+2i+2/2}\}$, respectively. Hence, $\eta_{v_i v_{i+1}} = (2mn - n + 2m - 2)/2$ and $\eta_{v_{i+1} v_i} = (n + 2)/2$. Therefore, using equation (3), $Mo(v_i v_{i+1}) = mn - n + m - 2$.

Case (iv) (for $xy \in E_4(J_{m,n})$): let $x = v_{(n-2/2)}$ and $y = v_{n/2}$, by means of Figure 1, we obtain $\eta(v_{(n-2/2)}, v_{n/2}) = \{u, w_1, w_3, \ldots, w_m, v_1, \ldots, v_{(n-2/2)}, v_{(3n+2/2)}, \ldots, v_{mn}\}$ and $\eta(v_{n/2}, v_{(n-2/2)}) = \{v_{n/2}, \ldots, v_n\}$. Therefore, $\eta_{v_{(n-2/2)} v_{n/2}} = mn - n + m - 1$ and $\eta_{v_{n/2} v_{(n-2/2)}} = (n + 2/2)$. Hence, using equation (3), $Mo(v_{(n-2/2)} v_{n/2}) = (2mn + 2m - 3n - 4)/2$.

Case (v) (for $xy \in E_5(J_{m,n})$): let $x = v_{n/2}$ and $y = v(n + 2/2)$, the resolving neighborhoods of $(v_{n/2}, v_{(n+2/2)})$ and $(v_{(n+2/2)}, v_{n/2})$ are $\{w_1, v_1, \ldots, v_{n/2}, v_{n(2m-1)/2} \ldots, v_{mn}\}$ and $\{w_2, v_{(n+2/2)}, \ldots, v_{(3n+2/2)}\}$, respectively. Hence, $\eta_{v_{(n/2)} v_{(n+2/2)}} = n + 2$ and $\eta_{v_{(n+2/2)} v_{n/2}} = n + 2$. Therefore, in view of equation (3), $Mo(v_{(n+2/2)} v_{n/2}) = 0$. □

Lemma 2. *For $xy \in E(J_{n,m})$ where n is odd and $n, m \ge 5$, then*

$$Mo(xy) = \begin{cases} mn - 2n + m - 3, & \text{if } xy \in E_1'(J_{n,m}), \\ mn - n + m - 2, & \text{if } xy \in E_2'(J_{n,m}), \\ mn - n + m - 2, & \text{if } xy \in E_3'(J_{n,m}), \\ mn - 2n + m - 3, & \text{if } xy \in E_4'(J_{n,m}). \end{cases} \quad (9)$$

Proof. The symmetry of the Jahangir graph $J_{n,m}$ allows us to compute the Mostar index of the following type of edges:

Case (i) (for $xy \in E_1(J_{m,n})$): let $x = u$ and $y = w_1$, the resolving neighborhoods of (u, w_1) and (w_1, u) are $\{u, w_2, \ldots, w_m, v_{(n+3)/2}, \ldots v_{mn-(n+1)/2}\}$ and $\{w_1, v_1, \ldots, v_{(n+1)/2}, v_{mn-(n-1)/2}-, \ldots, v_{mn}\}$, respectively. Hence, $\eta_{uw_1} = mn - n + m - 1$ and $\eta_{w_1u} = n + 2$. Therefore, applying equation (3), $Mo(uw_1) = mn - 2n + m - 3$.

Case (ii) (for $xy \in E_2(J_{m,n})$): let $x = v_1$ and $y = w_1$, by means of Figure 1, $\eta(v_1, w_1) = \{v_1, \ldots, v_{(n+3)/2}\}$ and $\eta(w_1, v_1) = \{u, w_1, \ldots, w_m, v_{(n+5)/2}, \ldots, v_{mn}\}$. Therefore, $\eta_{v_1w_1} = (n + 3)/2$ and $\eta_{w_1v_1} = (2mn - n + 2m - 1)/2$. Hence, using equation (3), $Mo(v_1w_1) = mn + m - n - 2$.

Case (iii) (for $xy \in E_3(J_{m,n})$): let $x = v_i$ and $y = v_{i+1}$ for $1 \le i \le (n-4/2)$, the resolving neighborhoods of (v_i, v_{i+1}) and (v_{i+1}, v_i) are $\{u, w_1, \ldots, w_m, v_1, \ldots, v_i, v_{(n+2i+5)/2} \ldots, v_{mn}\}$ and $\{v_{i+1}, \ldots, v_{(n+2i+1)/2}\}$, respectively. Hence, $\eta_{v_iv_{i+1}} = (2mn - n + 2m - 1)/2$ and $\eta_{v_{i+1}v_i} = (n + 3)/2$. Therefore, using equation (3), $Mo(v_iv_{i+1}) = mn - n + m - 2$.

Case (iv) (for $xy \in E_4(J_{m,n})$): let $x = v_{(n-1)/2}$ and $y = v_{(n+1)/2}$, the resolving neighborhoods of $(v_{(n-1)/2}, v_{(n+1)/2})$ and $\eta(v_{(n+1)/2}, v_{(n-1)/2})$ are $\{u, w_1, w_3, \ldots, w_m, v_1, \ldots, v_{(n-1)/2}, v_{3(n+1)/2}, \ldots, v_{mn}\}$ and $\{w_1, v_{(n+1)/2}, \ldots, v_{(3n+1)/2}\}$, respectively. Therefore, $\eta_{v_{(n-1)/2}v_{(n+1)/2}} = mn - n + m - 1$ and $\eta_{v_{(n+1)/2}v_{(n-1)/2}} = n + 2$. Hence, in view of equation (3), $Mo(v_{(n-1)/2}v_{(n+1)/2}) = mn - 2n + m - 3$. □

In the following theorem, the Mostar index of the Jahangir graph $J_{n,m}$ for $n, m \ge 5$ is computed.

Theorem 2. *For $xy \in E(J_{n,m})$, then*

$$Mo(J_{n,m}) = \begin{cases} m^2n^2 + 2m^2n - mn^2 + m^2 - 5mn - m, & \text{if } n = \text{even}, \\ m^2n^2 + 2m^2n - mn^2 + m^2 - 6mn - 5m, & \text{if } n = \text{odd}. \end{cases} \quad (10)$$

Proof.

Case (i) n = even: in view of Lemma 1 and equation (2), we have

$$\begin{aligned} Mo(J_{n,m}) &= \sum_{xy \in E_1(J_{n,m})} |n_{xy} - n_{yx}| + \sum_{xy \in E_2(J_{n,m})} |n_{xy} - n_{yx}| \\ &\quad + \sum_{xy \in E_3(J_{n,m})} |n_{xy} - n_{yx}| + \sum_{xy \in E_4(J_{n,m})} |n_{xy} - n_{yx}| \\ &\quad + \sum_{xy \in E_5(J_{n,m})} |n_{xy} - n_{yx}| \\ &= m(mn - 2n + m - 3) + 2m(mn + m - n - 1) - \\ &\quad + m(n-4)(mn - n + m - 2) + m(2mn + 2m - 3n - 4) + 0, \\ &= m^2n^2 + 2m^2n - mn^2 + m^2 - 5mn - m. \end{aligned} \quad (11)$$

Case (ii) n = odd: it is easy to see from Lemma 2 and equation (2) that

$$
\begin{aligned}
Mo\left(J_{n,m}\right) &= \sum_{xy\in E_1'(J_{n,m})} \left|n_{xy} - n_{yx}\right| + \sum_{xyE_2'(J_{n,m})} \left|n_{xy} - n_{yx}\right| \\
&\quad + \sum_{xy\in E_3'(J_{n,m})} \left|n_{xy} - n_{yx}\right| + \sum_{xy\in E_4'(J_{n,m})} \left|n_{xy} - n_{yx}\right|. \\
&= m(mn - 2n + m - 3) + 2m(mn + m - n - 2) \\
&\quad + m(n-3)(mn - n + m - 2) + 2m(mn - 2n + m - 3), \\
&= m^2n^2 + 2m^2n - mn^2
\end{aligned}
\tag{12}
$$

□

Here, we present a numerical example in support of the above theorem.

Example 1. Consider the Jahangir graph $J_{5,5}$; then, the sets $E_1'(J_{5,5}) = \{uw_1, uw_2, \ldots, uw_5\}$, $E_2'(J_{5,5}) = \{w_1v_{25}, w_2v_5, \ldots, w_5v_{21}\}$, $E_3'(J_{5,5}) = \{v_1v_2, v_4v_5, \ldots, v_{24}v_{25}\}$, and $E_4'(J_{5,5}) = \{v_2v_3, v_3v_4, \ldots, v_{23}v_{24}\}$ form the edge partition of $J_{5,5}$. Now for respective edges xy, st, uv, and pq in these partitions, Lemma 1 implies that $\eta_{xy} = 24$, $\eta_{yx} = 7$, and $Mo(xy) = 17$; $\eta_{st} = 27$, $\eta_{ts} = 4$, and $Mo(st) = 23$; $\eta_{uv} = 27$, $\eta_{vu} = 4$, and $Mo(uv) = 23$; $\eta_{pq} = 24$, $\eta_{qp} = 7$, and $Mo(pq) = 17$. Therefore, using Theorem 2, $Mo(J_{5,5}) = 5\times 17 + 2\times 5\times 23 + 5\times (5-4)\times 23 + 2\times 5\times 17 = 600$.

3. The Mostar Index of Cycle with a Chord Graph

In this section, the Mostar index of a cycle with a chord graph $C_{n,t}$ is computed. The graph $C_{n,t}$ is obtained from a cycle C_n by joining its two vertices at a distance $t-1$ as shown in Figure 2. The vertex set and edge set of $C_{n,t}$ are $V(C_{n,t}) = \{v_1, v_2, \ldots, v_n\}$ and $E(C_{n,t}) = \{v_iv_{i+1} | 1 \le i \le n\} \cup \{v_1v_t | 3 \le t \le (n+3)/2\}$, respectively, where subscripts are taken as mod n.

The edge set of $C_{n,t}$ is partitioned as follows.

For n = odd and t = even,

$$
\begin{aligned}
E_1^1\left(C_{n,t}\right) &= \left\{v_{t/2}v_{(t+2)/2}\right\}, \\
E_2^1\left(C_{n,t}\right) &= \left\{v_iv_{i+1} | 1 \le i \le v_{(t-2)/2}, v_{(t+2)/2} \le i \le t-1\right\}, \\
E_3^1\left(C_{n,t}\right) &= \left\{v_{(n+t-1)/2}v_{(n+t+1)/2}, v_{(n+t+1)/2}v_{(n+t+3)/2}\right\}, \\
E_4^1\left(C_{n,t}\right) &= \left\{v_{(n+t-3)/2}v_{(n+t-1)/2}, v_{(n+t+3)/2}v_{(n+t+5)/2}\right\}, \\
E_5^1\left(C_{n,t}\right) &= \left\{v_iv_{i+1} | t \le i \le \frac{n+t-5}{2}, \frac{n+t+5}{2} \le i \le n\right\}, \\
E_6^1\left(C_{n,t}\right) &= \left\{v_1v_t\right\}.
\end{aligned}
\tag{13}
$$

For n = odd, t = odd:

$$
\begin{aligned}
E_1^2\left(C_{n,t}\right) &= \left\{v_{t/2}v_{(t+2)/2}\right\}, \\
E_2^2\left(C_{n,t}\right) &= \left\{v_iv_{i+1} | 1 \le i \le \frac{t-3}{2}, \frac{t+3}{2} \le i \le t-1\right\}, \\
E_3^2\left(C_{n,t}\right) &= \left\{v_{(n+t)/2}v_{(n+t+2)/2}\right\}, \\
E_4^2\left(C_{n,t}\right) &= \left\{v_{(n+t-2)/2}v_{(n+t)/2}, v_{(n+t+2)/2}v_{(n+t+4)/2}\right\}, \\
E_5^2\left(C_{n,t}\right) &= \left\{v_iv_{i+1} | t \le i \le \frac{n+t-4}{2}, \frac{n+t+4}{2} \le i \le n\right\}, \\
E_6^2\left(C_{n,t}\right) &= \left\{v_1v_t\right\}.
\end{aligned}
\tag{14}
$$

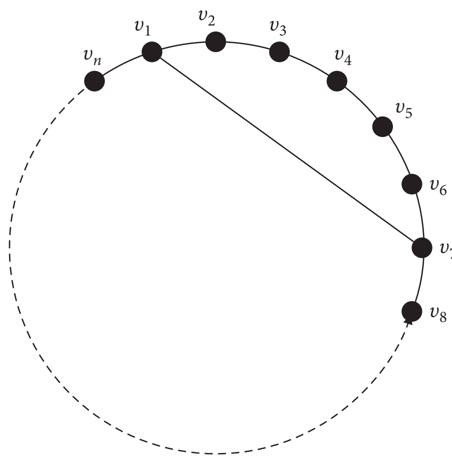

FIGURE 2: The cycle with a chord graph $C_{n,7}$.

For n = even and t = even,

$$\begin{aligned}
&E_1^3(C_{n,t}) = \{v_{t/2}v_{(t+2)/2}\},\\
&E_2^3(C_{n,t}) = \left\{v_iv_{i+1}|1\le i\le \frac{t-2}{2}, \frac{t+2}{2}\le i\le t-1\right\},\\
&E_3^3(C_{n,t}) = \{v_{(n+t)/2}v_{(n+t+2)/2}\},\\
&E_4^3(C_{n,t}) = \{v_{(n+t-2)/2}v_{(n+t)/2}, v_{(n+t)/2}v_{(n+t+2)/2}\},\\
&E_5^3(C_{n,t}) = \left\{v_iv_{i+1}|t\le i\le \frac{n+t-4}{2}, \frac{n+t+2}{2}\le i\le n\right\},\\
&E_6^3(C_{n,t}) = \{v_1v_t\}.
\end{aligned}\tag{15}$$

For n = even and t = odd,

$$\begin{aligned}
&E_1^4(C_{n,t}) = \{v_{(t-1)/2}v_{(t+1)/2}, v_{(t+1)/2}v_{(t+3)/2}\},\\
&E_2^4(C_{n,t}) = \left\{v_iv_{i+1}|1\le i\le \frac{t-3}{2}, \frac{t+3}{2}\le i\le t-1\right\},\\
&E_3^4(C_{n,t}) = \{v_{(n+t-1)/2}v_{(n+t+1)/2}, v_{(n+t+1)/2}v_{(n+t+3)/2}\},\\
&E_4^4(C_{n,t}) = \{v_{(n+t-3)/2}v_{(n+t-1)/2}, v_{(n+t+3)/2}v_{(n+t+5)/2}\},\\
&E_5^4(C_{n,t}) = \left\{v_iv_{i+1}|t\le i\le \frac{n+t-4}{2}, \frac{n+t+2}{2}\le i\le n\right\},\\
&E_6^4(C_{n,t}) = \{v_1v_t\}.
\end{aligned}\tag{16}$$

\In the following lemmas, the Mostar index corresponding to equations (13)–(16) of $C_{n,t}$ for $n\ge 9$ and $t\ge 3$ is computed.

Lemma 3. *Let $xy\in E(C_{n,t})$ where n is odd and t is even and $4\le t<(n+3)/2$, then*

$$Mo(xy) = \begin{cases} 0, & \text{if } xy\in E_1^1(C_{n,t}),\\ n-t, & \text{if } xy\in E_2^1(C_{n,t}),\\ \frac{t-2}{2}, & \text{if } xy\in E_3^1(C_{n,t}),\\ t-2, & \text{if } xy\in E_4^1(C_{n,t}),\\ t-2, & \text{if } xy\in E_5^1(C_{n,t}),\\ 0, & \text{if } xy\in E_6^1(C_{n,t}). \end{cases}\tag{17}$$

Proof.

Case (i) (for $xy\in E_1^1(C_{n,t})$): let $x=v_{t/2}$ and $y=v_{t+2/2}$, the resolving neighborhoods of $(v_{t/2}, v_{t+2/2})$ and $(v_{t+2/2}, v_{t/2})$ are $\{v_1,\ldots,v_{t/2}, v_{(n+t+3)/2},\ldots,v_n\}$ and $\{v_{t+2/2},\ldots,v_{n+t-1/2}\}$, respectively. Hence, $\eta_{v_{t/2}v_{t+2/2}} = \eta_{v_{t+2/2}v_{t/2}} = (n-1/2)$. Therefore, in view of equation (3), $Mo(v_{t/2}v_{t+2/2}) = 0$.

Case (ii) (for $xy\in E_2^1(C_{n,t})$): let $x=v_i$ and $y=v_{i+1}$ for $1\le i\le (t-2)/2$, by means of Figure 2, $\eta(v_i, v_{i+1}) = \{v_1,\ldots,v_i, v_{(t+2i+2)/2},\ldots,v_n\}$ and $\eta(v_{i+1}, v_i) = \{v_{i+1},\ldots,v_{(t+2i)/2}\}$. Hence, $\eta_{v_iv_{i+1}} = (2n-t)/2$ and $\eta_{v_{i+1}v_i} = (t/2)$. Therefore, equation (3) implies $Mo(v_iv_{i+1}) = n-t$.

Case (iii) (for $xy\in E_3^1(C_{n,t})$): let $x=v_{(n+t-1)/2}$ and $y=v_{(n+t+1)/2}$, then $\eta(v_{(n+t-1)/2}, v_{(n+t+1)/2}) = \{v_{(t+2)/2},\ldots,v_{(n+t-1)/2}\}$ and $\eta(v_{(n+t+1)/2}, v_{(n+t-1)/2}) = \{v_{(n+t-1)/2},\ldots,v_n\}$. Hence, $\eta_{v_{(n+t-1)/2}v_{(n+t+1)/2}} = (n-1)/2$ and

$\eta_{v_{(n+t+1)/2}v_{(n+t-1)/2}} = (n-t+1)/2$. Therefore, applying equation (3), $Mo(v_{(n+t-1)/2}v_{(n+t+1)/2}) = (t-2)/2$.

Case (iv) (for $xy \in E_4^1(C_{n,t})$): let $x = v_{(n+t-3)/2}$ and $y = v_{(n+t-1)/2}$, the resolving neighborhoods of $(v_{(n+t-3)/2}, v_{(n+t-1)/2})$ and $(v_{(n+t-1)/2}, v_{(n+t-3)/2})$ are $\{v_1, \ldots, v_{(n+t-3)/2}\}$ and $\{v_{(n+t-1)/2}, \ldots, v_{n-1}\}$, respectively. Therefore, $\eta_{v_{(n+t-3)/2}v_{(n+t-1)/2}} = (n+t-3)/2$ and $\eta_{v_{(n+t-1)/2}v_{(n+t-3)/2}} = (n+t-1)/2$. Hence, using equation (3), $Mo(v_{(n+t-3)/2}v_{(n+t-1)/2}) = t-2$.

Case (v) (for $xy \in E_5^1(C_{n,t})$): let $x = v_i$ and $y = v_{i+1}$ for $t \le i \le (n+t-5)/2$, the resolving neighborhoods of (v_i, v_{i+1}) and (v_{i+1}, v_i) are $\{v_1, \ldots, v_i, v_{(n+t+2i+4)/2}, \ldots, v_n\}$ and $\{v_{i+1}, \ldots, v_{(n-t+2i+2)/2}\}$, respectively. Hence, $\eta_{v_iv_{i+1}} = (n+t-2)/2$ and $\eta_{v_{i+1}v_i} = (n-t+2)/2$. Therefore, in view of equation (3), $Mo(v_iv_{i+1}) = t-2$.

Case (vi) (for $xy \in E_6^1(C_{n,t})$): let $x = v_1$ and $y = v_t$, then we have $\eta(v_1, v_t) = \{v_1, \ldots, v_{t/2}, v_{(n+t+3)/2}, \ldots, v_n\}$ and $\eta(v_t, v_1) = \{v_{(t+2)/2}, \ldots, v_{(n+t-1)/2}\}$. Hence, $\eta_{v_1v_t} = \eta_{v_tv_1} = (n-1)/2$. Therefore, using equation (3), $Mo(v_1v_t) = 0$. □

Lemma 4. *Let $xy \in E(C_{n,t})$ where n and t are odd and $3 \le t \le (n+1)/2$, then*

$$Mo(xy) = \begin{cases} \dfrac{n-t}{2}, & \text{if } xy \in E_1^2(C_{n,t}), \\ n-t, & \text{if } xy \in E_2^2(C_{n,t}), \\ 0, & \text{if } xy \in E_3^2(C_{n,t}), \\ t-2, & \text{if } xy \in E_4^2(C_{n,t}), \\ t-2, & \text{if } xy \in E_5^2(C_{n,t}), \\ 0, & \text{if } xy \in E_6^2(C_{n,t}). \end{cases} \quad (18)$$

Proof.

Case (i) (for $xy \in E_1^2(C_{n,t})$): let $x = v_{(t-1)/2}$ and $y = v_{(t-1)/2}$, the resolving neighborhoods of $(v_{(t-1)/2}, v_{(t+1)/2})$ and $(v_{(t+1)/2}, v_{(t-1)/2})$ are $\{v_1, \ldots, v_{(t-1)/2}, v_{(n+t+2)/2}, \ldots, v_n\}$ and $\{v_{(t-1)/2}, \ldots, v_{t-1}\}$, respectively. Hence, $\eta_{v_{(t-1)/2}v_{(t+1)/2}} = (n-1)/2$ and $\eta v_{(t+1)/2}v_{(t-1)/2} = (t-1)/2$. Therefore, using equation (3), $Mo(v_{(t-1)/2}v_{(t+1)/2}) = (n-t)/2$.

Case (ii) (for $xy \in E_2^2(C_{n,t})$): let $x = v_i$ and $y = v_{i+1}$ for $1 \le i \le (t-3)/2$, by means of Figure 2, $\eta(v_i, v_{i+1}) = \{v_1, \ldots, v_i, v_{(t+2i+3)/2}, \ldots, v_n\}$ and $\eta(v_{i+1}, v_i) = \{v_{i+1}, \ldots, v_{(t+2i-1)/2}\}$. Hence, $\eta_{v_iv_{i+1}} = (2n-t-1)/2$ and $\eta_{v_{i+1}v_i} = (t-1)/2$. Therefore, in view of equation (3), $Mo(v_iv_{i+1}) = n-t$.

Case (iii) (for $xy \in E_3^2(C_{n,t})$): let $x = (n+t)/2$ and $y = v_{(n+t+2)/2}$, then $\eta(v_{(n+t)/2}, v_{(n+t+2)/2}) = \{v_1, \ldots, v_{(t-1)/2}, v_{(n+t+2)/2}, \ldots, v_n\}$ and $\eta(v_{(n+t+2)/2}, v_{(n+t)/2}) = \{v_{(t+3)/2}, \ldots, v_{(n+t)/2}\}$. Hence, $\eta_{v_{(n+t)/2}v_{(n+t+2)/2}} = (n-1)/2$ and $\eta_{v_{(n+t+2)/2}v_{(n+t)/2}} = (n-1)/2$. Hence, equation (3) implies $Mo(v_{(n+t)/2}v_{(n+t+2)/2}) = 0$.

Case (iv) (for $xy \in E_4^2(C_{n,t})$): let $x = v_{(n+t-2)/2}$ and $y = v_{(n+t)/2}$, the resolving neighborhoods of $(v_{(n+t-2)/2}, v_{(n+t)/2})$ and $(v_{(n+t)/2}, v_{(n+t-2)/2})$ are $\{v_1, \ldots, v_{(n+t-2)/2}\}$ and $\{v_{(n+t)/2}, \ldots, v_n\}$, respectively. Therefore, $\eta_{v_{(n+t-2)/2}v_{(n+t)/2}} = (n+t-2)/2$ and $\eta_{v_{(n+t)/2}v_{(n+t-2)/2}} = (n-t+2)/2$. Henceforth, applying equation (3), $Mo(v_{(n+t-2)/2}v_{(n+t)/2}) = t-2$.

Case (v) (for $xy \in E_5^2(C_{n,t})$): let $x = v_i$ and $y = v_{i+1}$ where $t \le i \le (n+t-4)/2$, the resolving neighborhoods of v_iv_{i+1} and $v_{i+1}v_i$ are $\{v_1, \ldots, v_i, v_{(n-t+2i+4)/2}, \ldots, v_n\}$ and $\{v_{i+1}, \ldots, v_{(n-t+2i+2)/2}\}$, respectively. Hence, $\eta_{v_iv_{i+1}} = (n+t-2)/2$ and $\eta_{v_{i+1}v_i} = (n-t+2)/2$. Therefore, using equation (3), $Mo(v_iv_{i+1}) = t-2$.

Case (vi) (for $xy \in E_6^2(C_{n,t})$): let $x = v_1$, $y = v_t$, the resolving neighborhoods of (v_1, v_t) and (v_t, v_1) are $\{v_1, \ldots, v_{(t-1)/2}, v_{(n+t+2)/2}, \ldots, v_n\}$ and $\{v_{(t+3)/2}, \ldots, v_{(n+t)/2}\}$ respectively. Hence, $\eta_{v_1v_t} = (n-1)/2$ and $\eta_{v_tv_1} = (n-1)/2$. Therefore, equation (3) implies $Mo(v_1v_t) = 0$. □

Lemma 5. *Let $xy \in E(C_{n,t})$ where n and t are even and $4 \le t \le (n+2)/2$, then*

$$Mo(xy) = \begin{cases} 0, & \text{if } xy \in E_1^3(C_{n,t}), \\ n-t, & \text{if } xy \in E_2^3(C_{n,t}), \\ 0, & \text{if } xy \in E_3^3(C_{n,t}), \\ t-2, & \text{if } xy \in E_4^3(C_{n,t}), \\ t-2, & \text{if } xy \in E_5^3(C_{n,t}), \\ 0, & \text{if } xy \in E_6^3(C_{n,t}). \end{cases} \quad (19)$$

Proof.

Case (i) (for $xy \in E_1^3(C_{n,t})$): let $x = v_{t/2}$ and $y = v_{(t+2)/2}$, the resolving neighborhoods of $(v_{t/2}, v_{(t+1)/2})$ and $(v_{(t+1)/2}, v_{t/2})$ are $\{v_1, \ldots, v_{t/2}, v_{(n+t+2)/2}, \ldots, v_n\}$ and $\{v_{(t+2)/2}, \ldots, v_{(n+t)/2}\}$, respectively. Hence, $\eta_{v_{t/2}v_{(t+1)/2}} = n/2$ and $\eta_{v_{(t+1)/2}v_{t/2}} = n/2$. Therefore, using equation (3), $Mo(v_{t/2}v_{(t+2)/2}) = 0$.

Case (ii) (for $xy \in E_2^3(C_{n,t})$): let $x = v_i$ and $y = v_{i+1}$ for $1 \le i \le (t-2)/2$, the resolving neighborhoods of (v_i, v_{i+1}) and v_{i+1}, v_i are $\{v_1, \ldots, v_i, v_{(t+2i+2)/2}, \ldots, v_n\}$ and $\{v_{i+1}, \ldots, v_{(t+2i)/2}\}$, respectively. Therefore, $\eta_{v_iv_{i+1}} =$

$(2n-t)/2$ and $\eta_{v_{i+1},v_i} = t/2$. Hence, equation (3) implies $Mo(v_iv_{i+1}) = n-t$.

Case (iii) (for $xy \in E_3^3(C_{n,t})$): let $x = v_{(n+t)/2}$ and $y = v_{(n+t+2)/2}$, by means of Figure 2, $\eta(v_{(n+t)/2}, v_{(n+t+2)/2}) = \{v_{(t+2)/2}, \ldots, v_{(n+t)/2}\}$ and $\eta(v_{(n+t+2)/2}, v_{(n+t)/2}) = \{v_1, \ldots, v_{t/2}, v_{(n+t+2)/2}, \ldots, v_n\}$. Therefore, $\eta_{v_{(n+t)/2}v_{(n+t+2)/2}} = n/2$ and $\eta_{v_{(n+t+2)/2}v_{(n+t)/2}} = n/2$. Hence, applying equation (3), $Mo(v_{(n+t)/2}v_{(n+t+2)/2}) = 0$.

Case (iv) (for $xy \in E_4^3(C_{n,t})$): let $x = v_{(n+t-2)/2}$ and $y = v_{(n+t)/2}$, then $\eta(v_{(n+t-2)/2}, v_{(n+t)/2}) = \{v_1, \ldots, v_{(n+t-2)/2}\}$ and $\eta(v_{(n+t)/2}, v_{(n+t-2)/2}) = \{v_{(n+t)/2}, \ldots, v_n\}$. Hence, $\eta_{v_{(n+t-2)/2}v_{(n+t)/2}} = (n+t-2)/2$ and $\eta_{v_{(n+t)/2}v_{(n+t-2)/2}} = (n-t+2)/2$. Therefore, in view of equation (3), $Mo(v_{(n+t-2)/2}v_{(n+t)/2}) = t-2$.

Case (v) (for $xy \in E_5^3(C_{n,t})$): let $x = v_i$ and $y = v_{i+1}$ for $t \le i \le (n+t-4)/2$, the resolving neighborhoods of (v_i, v_{i+1}) and (v_{i+1}, v_i) are $\{v_1, \ldots, v_i, v_{(n-t+2i+4)/2}, \ldots, v_n\}$ and $\{v_{i+1}, \ldots, v_{(n-t+2i+2)/2}\}$, respectively. Hence, $\eta_{v_iv_{i+1}} = (n+t-2)/2$ and $\eta_{v_{i+1}v_i} = (n-t+2)/2$. Therefore, $Mo(v_iv_{i+1}) = t-2$.

Case (vi) (for $xy \in E_6^3(C_{n,t})$): let $x = v_1$ and $y = v_t$, then resolving neighborhoods are $\eta(v_1, v_t) = \{v_1, \ldots, v_{t/2}, v_{(n+t+2)/2}, \ldots, v_n\}$ and $\eta(v_t, v_1) = \{v_{(t+2)/2}, \ldots, v_{(n+t)/2}\}$. Hence, $\eta_{v_1v_t} = n/2$ and $\eta_{v_tv_1} = n/2$. Therefore, using equation (3), $Mo(v_1v_t) = 0$. □

Lemma 6. *Let $xy \in E(C_{n,t})$ where n is even and t is odd and $3 \le t \le (n+2)/2$, then*

$$Mo(xy) = \begin{cases} \dfrac{n-t+1}{2}, & \text{if } xy \in E_1^4(C_{n,t}), \\ n-t, & \text{if } xy \in E_2^4(C_{n,t}), \\ \dfrac{t-1}{2}, & \text{if } xy \in E_3^4(C_{n,t}), \\ t-2, & \text{if } xy \in E_4^4(C_{n,t}), \\ n-t-3, & \text{if } xy \in E_5^4(C_{n,t}), \\ 0, & \text{if } xy \in E_6^4(C_{n,t}). \end{cases} \tag{20}$$

Proof.

Case (i) (for $xy \in E_1^4(C_{n,t})$): let $x = v_{(t-1)/2}$ and $y = v_{(t+1)/2}$, the resolving neighborhoods of $(v_{(t-1)/2}, v_{(t+1)/2})$ and $(v_{(t+1)/2}, v_{(t-1)/2})$ are $\{v_1, \ldots, v_{(t-1)/2}, v_{(n+t+1)/2} \ldots, v_n\}$ and $\{v_{(t+1)/2}, \ldots, v_{t-1}\}$, respectively. Hence, $\eta_{v_{(t-1)/2}v_{(t+1)/2}} = n/2$ and $\eta_{v_{(t+1)/2}v_{(t-1)/2}} = (t-1)/2$. Therefore, using equation (3), $Mo(v_{(t-1)/2}v_{(t+1)/2}) = 2(n-t+1)/2 = (n-t+1)/2$.

Case (ii) (for $xy \in E_2^4(C_{n,t})$): let $x = v_i$ and $y = v_{i+1}$ for $1 \le i \le (t-3)/2$, by means of Figure 2, $\eta(v_i, v_{i+1}) = \{v_1, \ldots, v_i, v_{(t+2i+3)/2}, \ldots, v_n\}$ and $\eta(v_{i+1}, v_i) = \{v_{i+1}, \ldots, v_{(t+2i-1)/2}\}$. Hence, $\eta_{v_iv_{i+1}} = (2n-t-1)/2$ and $\eta_{v_{i+1}v_i} = (t-1)/2$. Therefore, in view of equation (3), $Mo(v_iv_{i+1}) = n-t$.

Case (iii) (for $xy \in E_3^4(C_{n,t})$): let $x = v_{(n+t-1)/2}$ and $y = v_{(n+t-1)/2}$, the resolving neighborhoods of $(v_{(n+t-1)/2}, v_{(n+t+1)/2})$ and $(v_{(n+t-1)/2}, v_{(n+t+1)/2})$ are $\{v_{(t+1)/2}, \ldots, v_{(n+t-1)/2}\}$ and $\{v_{(n+t+1)/2}, \ldots, v_n\}$, respectively. Therefore, $\eta_{v_{(n+t-1)/2}v_{(n+t+1)/2}} = n/2$ and $\eta_{v_{(n+t+1)/2}v_{(n+t-1)/2}} = (n-t+1)/2$. Hence, equation (3) implies $Mo(v_{(n+t-1)/2}v_{(n+t+1)/2}) = 2(t-1)/2 = (t-1)/2$.

Case (iv) (for $xy \in E_4^4(C_{n,t})$): let $x = v_{(n+t-3)/2}$ and $y = v_{(n+t-1)/2}$, we have $\eta(v_{(n+t-3)/2}, v_{(n+t-1)/2}) = \{v_1, \ldots, v_{(n+t-3)/2}\}$ and $\eta(v_{(n+t-1)/2}, v_{(n+t-3)/2}) = \{v_{(n+t-1)/2}, \ldots, v_{n-1}\}$. Hence, $\eta_{v_{(n+t-3)/2}v_{(n+t-1)/2}} = (n+t-3)/2$ and $\eta_{v_{(n+t-1)/2}v_{(n+t-3)/2}} = (n-t+1)/2$. Therefore, applying equation (3), $Mo(v_{(n+t-3)/2}v_{(n+t-1)/2}) = t-2$.

Case (v) (for $xy \in E_5^4(C_{n,t})$): let $x = v_i$ and $y = v_{i+1}$ where $t \le i \le (n+t-5)/2$, the resolving neighborhoods of (v_i, v_{i+1}) and (v_{i+1}, v_i) are $\{v_1, \ldots, v_i, v_{(n-t+2i+5)/2}, \ldots, v_n\}$ and $\{v_{i+1}, \ldots, v_{(n-t+2i+1)/2}\}$, respectively. Hence, $\eta_{v_iv_{i+1}} = (n+t-3)/2$ and $\eta_{v_{i+1}v_i} = (n+t-1)/2$. Therefore, using equation (3), $Mo(v_iv_{i+1}) = t-2$.

Case (vi) (For $xy \in E_6^4(C_{n,t})$): Let $x = v_1$ and $y = v_t$, then resolving neighborhoods are $\eta(v_1, v_t) = \{v_1, \ldots, v_{t/2}, v_{(n+t+3)/2}, \ldots, v_n\}$ and $\eta(v_t, v_1) = \{v_{(t+3)/2}, \ldots, v_{(n+t-1)/2}\}$. Hence, $\eta_{v_1v_t} = (n-2)/2$ and $\eta_{v_tv_1} = (n-2)/2$. Therefore, in view of equation (3), $Mo(v_1v_t) = 0$. □

In the following theorem, the Mostar index of cycle with a chord graph $C_{n,t}$ is computed.

Theorem 3. *For $xy \in E(C_{n,t})$ where $n \ge 9$ and $t \ge 3$, then*

$$Mo(C_{n,t}) = \begin{cases} 2(t-2)(n-t), & \text{if } n = \text{odd}, \\ 2(n-t)(t-2), & \text{if } n = \text{even}, \ t = \text{odd}, \\ 2(n-t)(t-2)-2, & \text{if } n = \text{even}, \ t = \text{even}, \end{cases} \tag{21}$$

Proof.

Case (i) $n = \text{odd}$, $t = \text{odd}$: in view of Lemma 3 and equation (2), we have

$$\begin{aligned}
Mo\left(C_{n,t}\right) &= \sum_{xy\in E_1^1\left(C_{n,t}\right)} \left|n_{xy}-n_{yx}\right| + \sum_{xyE_2^1\left(C_{n,t}\right)} \left|n_{xy}-n_{yx}\right| \\
&\quad + \sum_{xy\in E_3^1\left(C_{n,t}\right)} \left|n_{xy}-n_{yx}\right| + \sum_{xy\in E_4^1\left(C_{n,t}\right)} \left|n_{xy}-n_{yx}\right| \\
&\quad + \sum_{xy\in E_5^1\left(C_{n,t}\right)} \left|n_{xy}-n_{yx}\right| + \sum_{xy\in E_6^1\left(C_{n,t}\right)} \left|n_{xy}-n_{yx}\right|, \\
&= 1\times 0 + (t-2)\times(n-t) + 2\times\frac{(t-2)}{2} + 2\times(t-2) \\
&\quad + (n-t-3)\times(t-2) + 1\times 0, \\
&= 2(n-t)(t-2).
\end{aligned} \tag{22}$$

Case (ii) n = odd, t = even: in view of Lemma 4 and equation (2), we have

$$\begin{aligned}
Mo\left(C_{n,t}\right) &= \sum_{xy\in E_1^2\left(C_{n,t}\right)} \left|n_{xy}-n_{yx}\right| + \sum_{xyE_2^2\left(C_{n,t}\right)} \left|n_{xy}-n_{yx}\right| \\
&\quad + \sum_{xy\in E_3^2\left(C_{n,t}\right)} \left|n_{xy}-n_{yx}\right| + \sum_{xy\in E_4^2\left(C_{n,t}\right)} \left|n_{xy}-n_{yx}\right| \\
&\quad + \sum_{xy\in E_5^2\left(C_{n,t}\right)} \left|n_{xy}-n_{yx}\right| + \sum_{xy\in E_6^2\left(C_{n,t}\right)} \left|n_{xy}-n_{yx}\right|, \\
&= 2\times\frac{n-t}{2} + (t-3)\times(n-t) + 1\times 0 + 2\times(t-2) \\
&\quad + (n-t-2)\times(t-2) + 1\times 0, \\
&= 2(n-t)(t-2).
\end{aligned} \tag{23}$$

Case (iii) n = even, t = odd: in view of Lemma 5 and equation (2), we have

$$\begin{aligned}
Mo\left(C_{n,t}\right) &= \sum_{xy\in E_1^3\left(C_{n,t}\right)} \left|n_{xy}-n_{yx}\right| + \sum_{xyE_2^3\left(C_{n,t}\right)} \left|n_{xy}-n_{yx}\right| \\
&\quad + \sum_{xy\in E_3^3\left(C_{n,t}\right)} \left|n_{xy}-n_{yx}\right| + \sum_{xy\in E_4^3\left(C_{n,t}\right)} \left|n_{xy}-n_{yx}\right| \\
&\quad + \sum_{xy\in E_5^3\left(C_{n,t}\right)} \left|n_{xy}-n_{yx}\right| + \sum_{xy\in E_6^3\left(C_{n,t}\right)} \left|n_{xy}-n_{yx}\right|, \\
&= 1\times 0 + (t-2)\times(n-t) + 2\times\frac{(t-2)}{2} + 2\times(t-2) \\
&\quad + (n-t-3)\times(t-2) + 1\times 0, \\
&= 2(n-t)(t-2).
\end{aligned} \tag{24}$$

Case (iv) n = even, t = even: by using Lemma 6 and equation (2),

$$\begin{aligned}
Mo(C_{n,t}) &= \sum_{xy\in E_1^4(C_{n,t})} |n_{xy} - n_{yx}| + \sum_{xyE_2^4(C_{n,t})} |n_{xy} - n_{yx}| \\
&\quad + \sum_{xy\in E_3^4(C_{n,t})} |n_{xy} - n_{yx}| + \sum_{xy\in E_4^4(C_{n,t})} |n_{xy} - n_{yx}| \\
&\quad + \sum_{xy\in E_5^4(C_{n,t})} |n_{xy} - n_{yx}| + \sum_{xy\in E_6^4(C_{n,t})} |n_{xy} - n_{yx}|, \\
&= 2\times\frac{n-t+1}{2} + (t-3)\times(n-t) + 2\times\frac{(t-1)}{2} + 2\times(t-2) \\
&\quad + (n-t-3)\times(t-2) + 1\times 0, \\
&= 2(n-t)(t-2) - 2.
\end{aligned} \tag{25}$$

□

Now, we present a numerical example in support of the above theorem.

Examples 2. Consider a cycle with a chord graph $C_{14,7}$; then, the sets $E_1^4(C_{14,7}) = \{v_3v_4, v_4v_5\}$, $E_2^4(C_{14,7}) = \{v_1v_2, v_2v_3, v_5v_6, v_6v_7\}$, $E_3^4(C_{14,7}) = \{v_{10}v_{11}, v_{11}v_{12}\}$, $E_4^4(C_{14,7}) = \{v_9v_{10}, v_{12}v_{13}\}$, $E_5^4(C_{14,7}) = \{v_7v_8, v_8v_9, v_{13}v_{14}, v_{14}v_1\}$, and $E_6^4(C_{14,7}) = \{v_1v_7\}$ form the edge partition of $C_{14,7}$. Now for respective edges xy, st, uv, pq, ef, and jk in these partitions, Lemma 6 implies that $\eta_{xy} = 7$, $\eta_{yx} = 3$, and $Mo(xy) = 4$; $\eta_{st} = 10$, $\eta_{ts} = 3$, and $Mo(st) = 7$; $\eta_{uv} = 7$, $\eta_{vu} = 4$, and $Mo(uv) = 3$; $\eta_{pq} = 9$, $\eta_{qp} = 4$, and $Mo(pq) = 5$; $\eta_{ef} = 9$, $\eta_{fe} = 4$, and $Mo(ef) = 5$; $\eta_{jk} = 6$, $\eta_{kj} = 6$ and $Mo(jk) = 0$. Therefore, using Theorem 3, $Mo(C_{14,7}) = 1\times 0 + 4\times 5 + 2\times 5 + 2\times 3 + 4\times 7 + 2\times 4 = 70$.

4. Conclusion

In this article, by considering cycle-related graphs, we give explicit expressions of the Mostar index of the Jahangir graph and the cycle with a chord graph. The paper is concluded by the following open problems:

Open Problem 1: compute the Mostar index of certain classes of cycle-related structures such as convex polytopes Q_n, R_n, and A_n

Open Problem 2: compute the Mostar index of some rotationally symmetric graphs and S_n

Data Availability

All the data are included within this article. However, the reader may contact the corresponding author for more details of the data.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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