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MODELLING THE DYNAMICS OF RADICALIZATION WITH GOVERNMENT INTERVENTION

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ABSTRACT: Radicalization is the process by which people come to adopt increasingly extreme political, social or religious ideologies. When radicalization leads to violence, radical thinking becomes a threat to national and international security. Prevention programs are part of an effort to combat violent extremism and terrorism. This type of initiative seek to prevent radicalization process from occurring and taking hold in the first place. In this paper we introduce a simple compartmental model suitable to describe prevention programs in marginalized population by incorporating government inclusivity. We calculate the basic reproduction number R_0. For $R_0 < 1$ the system has one globally asymptotically stable equilibrium where no indoctrinated and radicals are present. For $R_0 > 1$ the system has an additional equilibrium where indoctrinated and radicals are persistence to the population. A Lyapunov function is used to show that, for $R_0 > 1$, the persistence radical equilibrium is globally asymptotically stable. Numerical simulation of the model carried out showed that enhanced government inclusivity leads to a slower rate of transition to radical population.

Key Words: reproduction number, radicalization, government inclusivity, marginalization

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1. INTRODUCTION

Radicalization is a broad term that applies to threats emanating from range of organisations and movements that use violence to pursue ideological or political goals. Radicalization is not limited to actions within any single faith or community. In the last 20 years, the threat of terrorism emerging across the globe has grown. Radicalization can lead to violent extremism and therefore becoming a major concern for national and international security.

Current counter terrorism strategies fall into two categories that is law enforcement approach where violent extremist are investigated prosecuted and imprisoned or military approach where violent extremist are killed or captured on the battle field. These two approaches alone cannot break the cycle of violence, Selim[6]. The realization of the inadequacy of the counter terrorism approach has lead to different strategies, collectively known as Countering Violent Extremism(C.V.E). C.V.E is a collection of noncoercive activities whose aim is to intervene in an individual's path towards violent extremism, to interdict criminal activity and to re-integrate those convicted of criminal activity into society. C.V.E programs can be divided into three broad class, Halloran[11]:

1. Prevention programs, which seek to prevent the radicalization process from occurring and taking hold in the first place.
2. Disengagement programs, which attempts to stop or control radicalization as it is occurring.
3. De-radicalization programs, which attempts to alter an individual extremist beliefs and violent behaviours with the aim to integrate him into the society. This type of programs often target convicted terrorists.

According to Keohane [8], radicalization cycle is generally composed of four sequential steps: pre-radicalization, identification, indoctrination, and action. Each of this stages is unique and has no specific signatures. Individuals who begin this process do not necessarily pass through all the stages, many stops or abandon this process at different points, Keohane [8]. Individual who pass through this entire process are quite likely to be involved in the planning or implementation of a terrorist act.

Pre-Radicalization is the point of origin for individuals before they begin this progression. It is their life situation before they were exposed to adopt radical ideology as their own ideology. Majority of individuals involved in these plot began as "unremarkable" they had ordinary jobs, had lived ordinary lives and had little, if any criminal record. Self-Identification is the phase where individual, influenced by both internal and external factors begins to explore radical ideology, gradually gravitates away from their old identity and begin to associate themselves with like-minded individuals and adopt this ideology as their own. There can be many types of triggers that can serve as catalyst, for instance: Economic such as loosing a job or blocked mobility, Social e.g alienation, discrimination, racism-real or perceived, political such as international conflicts involving muslims, personal such as death of close family member. Indoctrination is the phase in which an individual progressively intensifies his beliefs, wholly adopt the radical ideology and conclude, without any question. This phase is typically facilitated and driven by a spiritual sanctioner. The radical group consists of individual who have accepted their individual duty to participate in jihad and designate themselves as holy worriers. While other phases of radicalization may take place gradually, over two to three years, this jihadization component can be very rapid, taking only few months, or even weeks to run its course. Castillo [1] suggest the reduction in recruitment pool as one of the most effective mechanism in reducing the stock of terror capital. Compartmental models on radicalization were studied by Galam et al [12] and by MCCluskey et al [5]. Clutterbuck [3] built on compartmental model introduced in MCCluskey [5] by adding a treatment compartment to test the effectiveness of de-radicalization programs in countering violent extremism. They found out that increasing the average prison sentence was a successful strategy to counter violent extremism. However, the degree of government support for these programs hinges on the efficacy and, unfortunately, indicators of success and measure of effectiveness remains elusive, Halloran[11].

Individuals from marginalized areas are vulnerable to recruitment into terrorism. Marginalization, specifically, economic leads to poverty, illiteracy and bad governance leading to emergence of terror groups, Moghaddan [10]. In this paper we introduce a simple compartmental model suitable to describe prevention of radicalization in marginalized population with government inclusivity incorporated. Government inclusivity is paramount as prevention to radicalization especially if it is precipitate by marginalization. The government inclusivity index $(0 \leq \sigma < 1)$ measures the level of inclusivity. Inclusivity index is obtained using indicators that reflect cultural norms, policies, laws, and institutional practices.

2. MODEL DESCRIPTION AND FORMULATION

We formulate a mathematical model in which the population of interest T(t), is subdivided into non-core class G(t) and core class C(t), which is the radical part of the society, Castillo et al [1]. The non-core group G(t) is the general population and at-risk individual within the general population. This stage is usually the source of recruitment pool. The core group C(t), consists of susceptible class S(t), Indoctrinated class I(t) and Radical class R(t). The susceptible group S(t) includes members of population who have not yet been converted into adopting the ideology but have began to explore and gravitate towards radical ideas. Indoctrinated class I(t) includes those who have been converted and have intensified their beliefs and reinforce their radical views. The radical group R(t) consists of individuals who have internalized extreme ideology and have accepted their duties as jihads or holy warriors. Thus

$$C(t) = S(t) + I(t) + R(t). \tag{1}$$

The considered total population is

$$T(t) = G(t) + C(t). \tag{2}$$

We define β_1, β_2 and β_3 as parameters, which describes the strength of recruitment from one subgroup to another. Therefore we are able to model dynamics inside the core:

$$\gamma_1 = \frac{\beta_1(1-\sigma)G(t)C(t)}{T(t)} \tag{3}$$

is the transition term from G(t) to S(t). The indoctrinated group has more chance of recruiting others since it is driven by a spiritual sanctioner, Cherif et al [4], therefore we model the dynamics between S(t) and I(t) and the transfer rate from I(t) to R(t) inside the core by;

$$\gamma_2 = \frac{\beta_2(1-\sigma)S(t)I(t)}{C(t)} \quad \text{and} \quad \gamma_3 = \beta_3(1-\sigma)I(t), \tag{4}$$

respectively. Here $0 \leq \sigma < 1$ is incorporated government inclusivity.

The the transfer diagram for this system is given below.

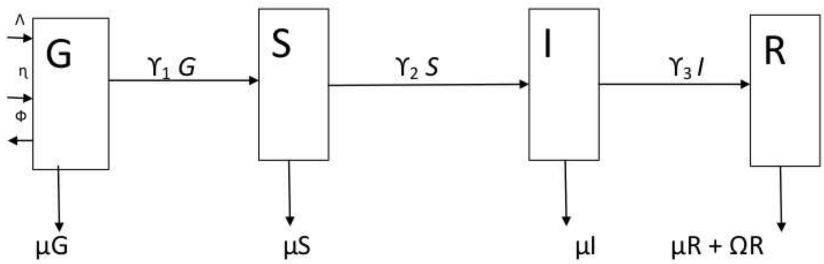

Figure 1: The transfer diagram for the radicalization model

Thus the radicalization model consists of the following differential equations together with non-negative conditions:

$$\begin{aligned}
\frac{dG}{dt} &= \Lambda + \eta - \phi - \frac{\beta_1(1-\sigma)G(t)C(t)}{T(t)} - \mu G(t), \\
\frac{dS}{dt} &= \frac{\beta_1(1-\sigma)G(t)C(t)}{T(t)} - \delta S(t) - \mu S(t), \\
\frac{dI}{dt} &= \delta S(t) - \gamma I(t) - \mu I(t),
\end{aligned}$$

$$\frac{dR}{dt} = \gamma I(t) - \mu R(t) - \Omega Rt. \tag{5}$$

where Λ is the birth rate, ϕ is the emigration rate, η is immigration rate $\delta = \beta_2(1 - \sigma)\frac{I(t)}{C(t)}$, $\gamma = \beta_3(1-\sigma)$ and $0 \leq \sigma < 1$ is the measure of government inclusivity.

3. SCALING THE MODEL

In this section our aim is to create a reduced system of (ODE_s) which is "dynamically" equivalent to the system (5). The dynamics such as the equilibria of the unscaled system and reduced system are same, Castillo [2]. Adding all equations in system (5) yields,

$$\frac{dT}{dt} = \Lambda + \eta - \phi - \mu T(t). \tag{6}$$

Solving (6) with initial value ($T_{t0} = T_0$) we have;

$$T(t) = \frac{\Lambda}{\mu} + \frac{\eta}{\mu} - \frac{\phi}{\mu} + \left(T_0 - \frac{\Lambda}{\mu} - \frac{\eta}{\mu} + \frac{\phi}{\mu}\right)e^{-\mu(t)}. \tag{7}$$

and thus

$$T(t)_{\lim t\to\infty} = \frac{\Lambda}{\mu} + \frac{\eta}{\mu} - \frac{\phi}{\mu} \tag{8}$$

Substituting (8) in (5) we obtain the following reduced systems of (ODE_S),

$$\begin{aligned} \frac{dS}{dt} &= r(1 - \frac{C(t)}{K})C(t) - \delta S(t) - \mu S(t) \\ \frac{dI}{dt} &= \delta S(t) - \gamma I(t) - \mu I(t) \\ \frac{dR}{dt} &= \gamma I(t) - (\mu + \Omega)R(t). \end{aligned} \tag{9}$$

Where $r = \beta_1(1-\sigma)$, $K = \frac{\Lambda}{\mu} + \frac{\eta}{\mu} - \frac{\phi}{\mu}$ and $C(t) = S(t) + I(t) + R(t)$. The transition from G(t) into C(t) is of logistic form. The core population increases strongly until it is stopped by exhaustive recruitment pool.

4. ANALYSIS OF THE MODEL

Since we are dealing with a human population, we expect that all population compartments be non negative $\forall t > 0$ in the feasible region Γ where $S(t), I(t), R(t) \in \Gamma \subset \mathcal{R}^3_+$. It can be shown that all solutions are bounded in Γ, $\forall t > 0$ such that $0 \leq C \leq \frac{r}{\mu}$. Thus the model is epidemiologically well posed in the region Γ and can be analysed.

4.1. RADICAL FREE EQUILIBRIUM (RFE) POINT

The Radical Free Equilibrium point (RPE) of equation (9) is obtained by setting the indoctrinated and radical classes to zero. This gives

$$\begin{aligned} 0 &= r(1-\frac{C(t)}{K})C(t)-\delta S(t)-\mu S(t),\\ 0 &= \delta S(t)-\gamma I(t)-\mu I(t),\\ 0 &= \gamma I(t)-(\mu+\Omega)R(t). \end{aligned} \tag{10}$$

At the Radical Free State (RFE), there are no radical ideologies that's $I=0, R=0$, and the system of equations (10) reduces to

$$0 = rC(1-\frac{C}{K})-\mu S. \tag{11}$$

Making S the subject of equation (11) results to

$$S = K(1-\frac{\mu}{r}).$$

The RFE point for equation (9) is given by,

$$E^0 = (K(1-\frac{\mu}{r}),0,0).$$

Which indicates that in absence of radical ideologies system of equations (9) will consist of only one compartment class(Susceptible).

4.2. THE BASIC REPRODUCTIVE NUMBER R_0

The basic reproduction number R_0 is the spectral radius of the next generation matrix calculated at RFE. R_0 can be calculated as follows (see Watmough et al [13] for more details). In our case the infected compartment is I since we assume that radicalized class has a low chance of recruiting susceptible population, Horgan [7].

Defination 4.2.1. The basic reproduction number R_0 is the average number of susceptible persons one Indoctrinated person can recruit in a susceptible population. If $R_0<1$ it means the radical ideologies will die out in the population and $R_0>1$ means the radical ideologies are persistence in the population. The basic reproductive number of the system is determined using next generation matrix approach, Watmough et al [13]. The basic reproductive number is important since it is directly related to the effort required to eliminate radical ideologies.

Lemma 4.2.1. *Basic reproduction number of the Model (9) is given by*

$$R_0 = \frac{\beta_2(1-\sigma)}{\beta_3(1-\sigma)+\mu}. \tag{12}$$

Proof. Consider that $\mathcal{F}_i$ is the rate of appearance of new radical in compartment associated with index i, $\mathcal{V}^{+}{}_i$ is the rate of transfer of individual into compartment associated with index i by all other means and $\mathcal{V}^{-}{}_i$ is the rate of transfer of individuals out of of compartment associated with index i. In this way, the matrices $\mathcal{F}_i$, $\mathcal{V}^{+}{}_i$ and $\mathcal{V}^{-}{}_i$ associated with model (9) are given by

$$\mathcal{F}_i = \begin{pmatrix} \delta S \\ 0 \end{pmatrix}, \mathcal{V}^{+}{}_i = \begin{pmatrix} 0 \\ \gamma I \end{pmatrix} \quad \text{and} \quad \mathcal{V}^{-}{}_i = \begin{pmatrix} \gamma I + \mu I \\ -\gamma I + \mu R + \Omega R \end{pmatrix} \tag{13}$$

Therefore considering $\mathcal{V}_i = \mathcal{V}^{-}{}_i - \mathcal{V}^{+}{}_i$ results to $[S'(t) + I'(t) + R'(t)]^T = \mathcal{F}_i - \mathcal{V}_i$. The jacobian of $\mathcal{F}_i$ and $\mathcal{V}_i$ evaluated at radical free equilibrium are respectively given by,

$$F = \begin{pmatrix} \beta_2(1-\sigma) & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad V = \begin{pmatrix} \gamma + \mu & 0 \\ -\gamma & \mu + \Omega \end{pmatrix}. \tag{14}$$

Determining the inverse of matrix V yields;

$$V^{-1} = \begin{pmatrix} \frac{1}{\gamma+\mu} & 0 \\ \frac{\gamma}{(\gamma+\mu)(\mu+\Omega)} & \frac{1}{\mu+\Omega} \end{pmatrix} \tag{15}$$

The basic reproduction number of equation (9) is the spectral radius of $\rho(FV^{-1})$ given by

$$R_0 = \rho(FV^{-1}) = \frac{\beta_2(1-\sigma)}{\beta_3(1-\sigma) + \mu}.$$

□

4.3. LOCAL STABILITY OF RADICAL FREE EQUILIBRIUM (RFE) POINT

The stability of an equilibrium point determines whether or not solutions nearly the equilibrium point remains nearby, get closer or get further a way.

Definition 4.3.1. For local stability, perturbing the RFE, the system stays in the neighborhood of equilibrium point or, for asymptotic stability, it returns to equilibrium point.

Theorem 4.3.1. *The RFE of the system of equations* (9) *is locally asymptotically stable whenever* $R_0 < 1$ *and unstable otherwise.*

Proof. From the system of equations (9), the Jacobian matrix evaluated at $E^0 = (S^0, I^0, R^0)$ is given by

$$J_{E^0} = \begin{pmatrix} (\mu - r) & \delta & 0 \\ 0 & (\gamma+\mu)(R_0 - 1) & 0 \\ 0 & \gamma & -(\mu+\Omega) \end{pmatrix}. \tag{16}$$

The stability of this steady states can then be determined based on the signs of eigenvalues of the corresponding Jacobian matrix. Solving for eigenvalues of matrix (16) yields $\lambda_1 = \mu - r$, $\lambda_2 = (\gamma + \mu)(R_0 - 1)$ and $\lambda_3 = -(\mu + \Omega)$. For local asymptotic stability, all real parts λ should be negative. λ_1 is negative since the population growth rate r is assumed to be greater than natural death rate μ for a growing population, λ_3 is negative and for λ_2 real part to be negative then $R_0 < 1$. Thus if $R_0 < 1$ then all the roots are negative and given that $R_0 < 1$, the radical free equilibrium state (RFE) of the model is asymptotically stable. □

4.4. GLOBAL STABILITY OF THE RADICAL FREE EQUILIBRIUM (RPE) POINT

Definition 4.4.1. An equilibrium is global (asymptotically) stable if it is the unique equilibrium of the dynamical system and the property hold globally (its domain of attraction is entire state space)

Theorem 4.4.1. *If $R_0 < 1$, then the radical free equilibrium point $E^0 = (S^0, I^0, R^0)$ of equation (9) is globally asymptotically stable in Γ and unstable if $R_0 > 1$*

Proof. To proof this the following Lyapunov function is used

$$L(S, I, R) = \beta_2(\mu + \Omega)I.$$

The lyapunov function $L(S, I, R)$ satisfies the conditions, $L(S^0, I^0, R^0) = 0$ and $L(S, I, R) > 0$, hence it is a positive definite. For

$$\frac{dL(S, I, R)}{dt} \tag{17}$$

To be negative definite, it must satisfy the conditions,

$$\frac{dL(S^0, I^0, R^0)}{dt} = 0, \quad \text{and} \quad \frac{dL(S, I, R)}{dt} < 0. \tag{18}$$

Determining the time derivative of the lyapunov equation yields ,

$$\frac{dL(S, I, R)}{dt} = \beta_2(\mu + \Omega)\frac{dI}{dt}.$$

Substituting for $\frac{dI}{dt}$ results to

$$\frac{dL(S, I, R)}{dt} = \beta_2(\mu + \Omega)[\beta_2(1 - \sigma)\frac{IS}{C} - \gamma I - \mu I].$$

Factor $(\gamma + \mu)$ to have

$$\frac{dL(S, I, R)}{dt} = \beta_2(\mu + \Omega)(\gamma + \mu)[R_0\frac{S}{C} - 1]I.$$

If $I = 0$ then $\frac{dL(S,I,R)}{dt} = 0$ but if $I \neq 0$ and $R_O < 1$ then $\frac{dL(S,I,R)}{dt} < 0$.

Therefore, the radical free equilibrium is globally asymptotically stable in some region Γ. This means the radical ideologies can be eradicated in finite time. □

4.5. LOCAL STABILITY OF THE RADICAL PERSISTENCE EQUILIBRIUM (RPE) POINT

The endemic equilibrium state is the state where the radical ideologies cannot be totally eradicated but remains in the population. For radical ideologies to persist in the population, the indoctrinated class, the susceptible class and radical class must not be zero at equilibrium point. In other words, if $E^*(S^*, I^*, R^*)$ is endemic equilibrium state, then $E^*(S^*, I^*, R^*) \neq (0, 0, 0)$

Theorem 4.5.1. *A positive radical persistent equilibrium point exists and is locally asymptotically stable whenever* $R_0 > 1$

Proof. In order to obtain the radical persistent equilibrium state, we solve equations $(22, 23, 24, 25)$ simultaneously taking into consideration the fact that (I and$R \neq 0$).

$$r(1 - \frac{S(t)}{K})S(t) - \delta S(t) - \mu S(t) = 0, \tag{19}$$

$$\delta S(t) - \gamma I(t) - \mu I(t) = 0, \tag{20}$$

$$\gamma I(t) - (\mu + \Omega)R(t) = 0. \tag{21}$$

For $I, R \neq 0$ and

$$rC(1 - \frac{C}{K}) - \mu C = 0.$$

From the equations above we have, for $I, R \neq 0$,

$$C^* = K(1 - \frac{\mu}{r}),$$

$$S^* = (\frac{1}{R_0})K(1 - \frac{\mu}{r}),$$

$$I^* = \frac{\mu}{\beta_2(1-\sigma)}(R_0 - 1)K(1 - \frac{\mu}{r}),$$

$$R^* = \frac{\mu\gamma}{\beta_2(\mu+\Omega)(1-\sigma)}(R_0 - 1)K(1 - \frac{\mu}{r}).$$

Which is positive provided $R_0 > 1$.

Therefore, positive radical persistent equilibrium point exist

$$E^* = (S^*, I^*, R^*) =$$

$$((\frac{1}{R_0})K(1 - \frac{\mu}{r}), \frac{\mu}{\beta_2(1-\sigma)}(R_0 - 1)K(1 - \frac{\mu}{r}), \frac{\mu\gamma}{\beta_2(\mu+\Omega)(1-\sigma)}(R_0 - 1)K(1 - \frac{\mu}{r}))$$

and is locally asymptotically stable whenever $R_0 > 1$. □

4.6. GLOBAL STABILITY OF RADICAL PERSISTENCE EQUILIBRIUM (RPE) POINT

Theorem 4.6.1. *Radical Persistence Equilibrium (RPE) point is stable whenever* $A > B$, *where* $A = \gamma\frac{(S-S^*)^2}{S} + \mu S^*\frac{S^*}{S} + \mu IS + \mu S\frac{I^*}{I} + \frac{\mu I(\gamma+\mu)\frac{R^*}{R}}{\delta} + \frac{\mu I(\gamma+\mu)}{\delta} + \frac{\mu(\gamma+\mu)}{\delta\gamma}(\mu+\Omega)R^*$ *and* $B = \mu S^* + \frac{\mu}{\delta}I^*(\gamma+\mu) + \frac{\mu(\gamma+\mu)}{\delta\gamma}(\mu+\Omega)R^* + \mu S + \frac{\mu I(\gamma+\mu)}{\delta}$

Proof. To proof this the following Lyapunov function is used

$$\frac{K(S,I,R)}{dt} = S - S^* - S^*ln\frac{S}{S^*} + X_1(I - I^* - I^*ln\frac{I}{I^*}) + X_2(R - R^* - R^*ln\frac{R}{R^*}),$$

where$X_1$and X_2 are positive constant to be determined. The Lyapunov function K(S,I,R) satisfies the conditions $K(S^*, I^*, R^*) = 0$ and $K(S, I, R) > 0$, hence positive definite. For $\frac{dK(SIR)}{dt}$ to be negative definite, it must satisfy

$$\frac{dH(S^*, I^*, R^*)}{dt} = 0 \quad \text{and} \quad \frac{dH(S, I, R)}{dt} < 0.$$

The radical persistence equilibrium point $E^* = (S^*, I^*, R^*)$ for system of equations (5) satisfies

$$\begin{aligned} C(1-\frac{C}{K}) &= \delta S^* + \mu S^* \\ \delta S^* &= (\gamma+\mu)I^* \\ \gamma I^* &= (\mu+\Omega)R^*. \end{aligned} \tag{22}$$

Determining the time derivative of Lyapunov function results to

$$\frac{dK(S,I,R)}{dt} = (1-\frac{S^*}{S})\frac{dS}{dt} + (1-\frac{I^*}{I})\frac{dI}{dt} + (1-\frac{R^*}{R})\frac{dR}{dt} \tag{23}$$

Substituting in $\frac{dS}{dt}$, $\frac{dI}{dt}$,$\frac{dR}{dt}$ together with equation (26) in (27) yields

$$\begin{aligned} \frac{dl(S,I,R)}{dt} &= (1-\frac{S^*}{S})(\delta S^* + \mu S^* - \delta S - \mu IS) + X_1(1-\frac{I^*}{I})(\delta S - \mu I - \gamma I) \\ &\quad + X_2(1-\frac{R^*}{R})(\gamma I - (\mu+\Omega)R) \end{aligned} \tag{24}$$

Upon expanding and simplifying equation (28) yields,

$$\begin{aligned} \frac{dL(S,I,R)}{dt} &= -\gamma\frac{(S-S^*)^2}{S} - \mu S^*\frac{S^*}{S} + \mu S^* - \mu IS + X_1(-\delta S\frac{I^*}{I} \\ &+ \gamma I^* + \mu I^*) + X_2(-\gamma I\frac{R^*}{R} + (\mu+\Omega)R^*) + X_1(\delta S - \gamma I - \mu I) + \\ &X_2(\gamma I - (\mu+\Omega)R^*) \end{aligned} \tag{25}$$

Setting SI and I to zero yields

$$-\mu IS + X_1\delta S = 0,$$

hence

$$X_1 = \frac{\mu}{\delta} - X_1 I(\gamma+\mu) + X_2\gamma I = 0,$$

hence $X_2 = \frac{\mu(\gamma+\mu)}{\delta\gamma}$. Substituting X_1 and X_2 in (29) and simplify result to

$$\begin{aligned}\frac{dL(S,I,R)}{dt} &= -[\gamma\frac{(S-S^*)^2}{S} + \mu S^*\frac{S^*}{S} + \mu IS + \mu S\frac{I^*}{I} + \frac{\mu I(\gamma+\mu)\frac{R^*}{R}}{\delta} \\ &\quad + \frac{\mu I(\gamma+\mu)}{\delta} + \frac{\mu(\gamma+\mu)}{\delta\gamma}(\mu+\Omega)R^*] + [\mu S^* + \frac{\mu}{\delta}I^*(\gamma+\mu) + \\ &\quad \frac{\mu(\gamma+\mu)}{\delta\gamma}(\mu+\Omega)R^* + \mu S + \frac{\mu I(\gamma+\mu)}{\delta}] \end{aligned} \tag{26}$$

If we set

$$\begin{aligned} A &= \gamma\frac{(S-S^*)^2}{S} \\ &+ \mu S^*\frac{S^*}{S} + \mu IS + \mu S\frac{I^*}{I} + \frac{\mu I(\gamma+\mu)\frac{R^*}{R}}{\delta} + \frac{\mu I(\gamma+\mu)}{\delta} + \frac{\mu(\gamma+\mu)}{\delta\gamma}(\mu+\Omega)R^* \end{aligned}$$

and

$$B = \mu S^* + \frac{\mu}{\delta}I^*(\gamma+\mu) + \frac{\mu(\gamma+\mu)}{\delta\gamma}(\mu+\Omega)R^* + \mu S + \frac{\mu I(\gamma+\mu)}{\delta}.$$

By inspection clearly $A > B$. Hence the radical persistence equilibrium is globally asymptotically stable implying that the radical ideologies transmission levels can be kept quite low in the presence of government inclusivity. □

4.7. NUMERICAL SIMULATION OF THE MODEL

Numerical simulation are carried out to graphically illustrate the long term effect of government inclusivity on the dynamics of radicalization.

5. DISCUSSION

First, we varies the rate of government inclusivity (σ) and leave the rest of the parameters unchanged to study the effect of government inclusivity on dynamics of radicalization. If $R_0 < 1$, $R_0 = 0.163265$, what this means is that, the radical free equilibrium state is asymptotically state. This can be viewed as an improvement of the prevention programs. Figure 3 shows that $S, I, R \longrightarrow 0$, as the time t grows large, confirming that E_0 is a global asymptotically stable. This is the preferred situation, where indoctrinated and radical die out in the long ran. When $R_0 > 1$, and thus by theorem 4.5.1, the radical persistence equilibrium E^* is globally asymptotically stable. Figures 2 depict indoctrinated and Radical classes as a function of time t(days), and

Table 1: Parameters values for radical model.

Description	Parameter	Range	Source
Natural mortality rate	(μ)	0.000034247	[9]
Radical induced death rate	(Ω)	0.0036	[9]
The level of government inclusivity	(σ)	0.34-0.92	[Varies]
intrinsic growth rate of core population	r	0.5	[1]
The indoctrination rate	(β_2)	$0.000005 day^{-1}$	[12]
Carrying capacity	K	100	[Assumed]
The self identification rate	(β_1)	$0.0035 day^{-1}$	[Assumed]
The radicalization rate	(β_3)	$0.00045 day^{-1}$	[Assumed]

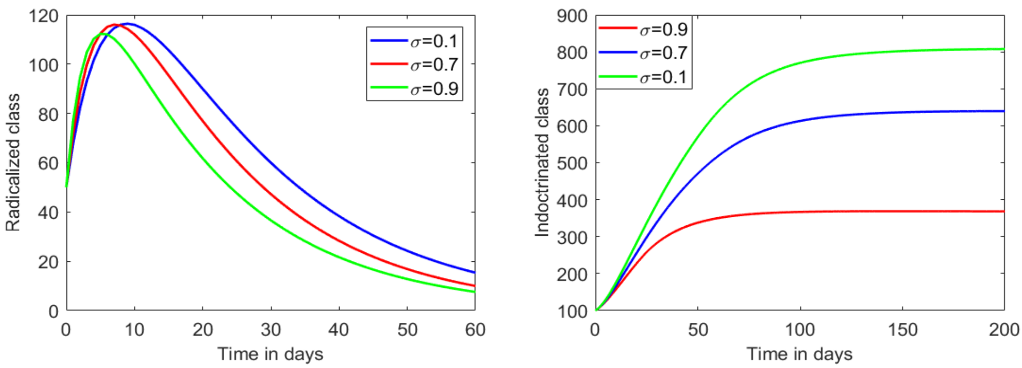

Figure 2: The size of the radical and indoctrinated class over time for the system (9) with different values of σ when $R_0 > 1$

shows that there is a change in these population after which it approaches a constant value. This case illustrates the unwanted scenario where indoctrinated and radical become persistence to the population. Government inclusivity leads to susceptible population shunning away from radical ideologies leading to reduction in the population of radicals (see figure 2). When modeling social dynamics one has to make many simplifying assumptions. The model studied in this paper is not completely from this defect. One issue is that the population in various compartment may not be homogeneous. For instance, the parameter β_1 may depend on the age of the non-core population, suggesting that an age-structured model may be better suited to describe this problem. Future studies should address this issue.

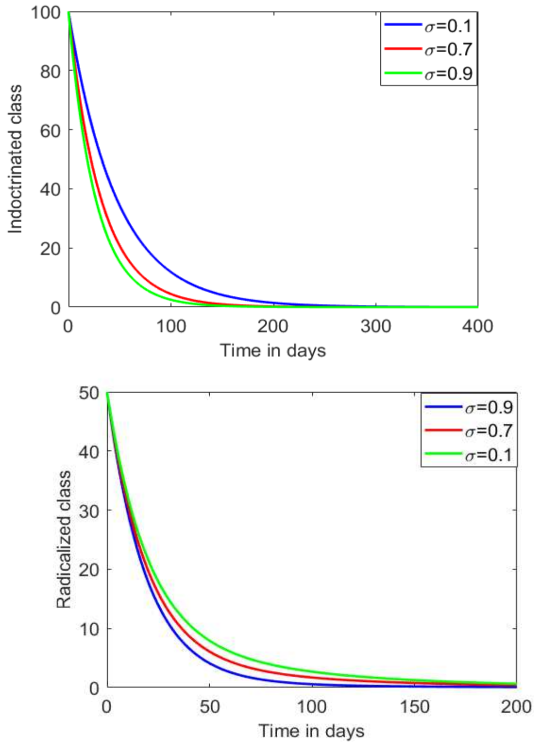

Figure 3: The size of the radical and indoctrinated class over time for the system (9) with different values of σ when $R_0 < 1$

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6. CONCLUSION

We conclude that effective government inclusivity will greatly reduce the number of people transiting from non-core to core. This will reduce the number of people radicalized in a given time. With government inclusivity the susceptible population will shun a way from radical ideologies leading to a very small fraction of individuals in a given population progressing to radical stage.

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