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Modeling fertility rate of Rwanda in the presence of interference: the 1994 genocide

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ABSTRACT

Modeling of fertility rate has attracted the interest of demographers for many years. However, very little has been documented in the existing literature concerning modeling of fertility in the presence of interference, yet interference to fertility is a common phenomenon. In this this study, we have modeled fertility rate in the presence of interference. We have fitted Probability distribution (alternative statistical models) to both interference free data set of Rwanda 1992 and to the Rwanda 2000 fertility data set which had interference in it, so as to determine the effect of interference on the shape of the fertility data. The parameter of the distributions were estimated and then fitted by use of the maximum likelihood estimation method available in the `fitdistriplus' package in the R statistical software. The model life table approach was also used to determine the net fertility value F0 which was modeled from the net reproduction value R0 .A relationship between fertility rate in the presence of interference and population growth was then established. A major finding of this study is that interference free fertility data fit a gamma distribution, data containing interference fit a modified gamma distribution with a mounded but skewed shape. The results also show in the presence of interference F0 >2 hence population increases.

1. Introduction

Fertility in this context refers to the actual production of children, and not the physical capability to produce them, which is termed as fecundity. Demographers have always measured how quickly a population is growing by determining how frequently people are added to the population by being born. This has been done by measuring fertility rate of a population. Barret, Bogue and Anderson in their study (Barret *et al.*, 1997), define ASFR as the annual number of births per woman in a particular age group expressed per 1ooo women in that age group. They refer to TFR as the average number of children that would be born to a woman over her reproductive lifetime if, she were to experience the exact current age specific fertility rates through her reproductive lifetime and also were to survive from birth through the end of her reproductive life. TFR is therefore, not only a more direct measure of the level of fertility, but also an indicator of the potential for population change in a country. A rate of two children per woman is considered to be the replacement rate for a population, leading to stability in terms of total numbers, a rate of above two children would mean that a population is growing in size while a rate of below two children would mean that a population is declining in size and growing older (Onoja & Osayomore, 2012). To measure TFR, Researchers consider the fact that fertility varies over the course of women's lifetime, and therefore, construct a measure based on the age-specific fertility rates observed in a given year (Barret *et al.*, 1997). Considering the Rwanda DHS data sets of the years 1992, 2000 and 2005 (RDHS, 1992; RDHS, 2000; RDHS, 2005), the analysis of the mean number of children born were 5.885, 6.330 and 5.876 in the respective years. We hypothesize that the 1994 Genocide interference must have been responsible for the high mean number of children born in the year 2000. The term Interference, in this context refers to a situation of large scale strike of unanticipated natural phenomenon such as high magnitude earthquake, major floods, Wars and Genocides, which leave many people dead and thousands displaced. Fertility however, is believed to be affected if a sudden interference occurs in a population. Following the experience of interference, households may have the incentive to increase the number of children ever born thereby leading to a positive fertility response in excess of replacement effects (Schultz, 1973). Many investigators have observed that in the event of interference mortality may be experienced and that among those who die during the event are children, a scenario which causes replacement intentions. Such intentions usually extend beyond those women whose children die and operates through social groups like the extended families and even ethnic groups thereby, causing a change in the fertility pattern\cite. The strike of interference to fertility may in a big way, change a household's preference for children through a changed perception of community spirit, or the return of the traditional family values, all which encourage replacement effects, which is consistent with response to child mortality (Schultz, 1973). Effects of large scale unanticipated phenomena have been observed at both aggregate and individual level. Many researchers have documented a fertility decline during the interference followed by fertility rise after the interference. For instance, Lindstrom and Berhanu, in their study concerning the specific impacts of conflict on fertility in Ethiopia documented a sharp temporary decline in fertility during the early years of violent conflict and famine, followed by a high rise in fertility thereafter (Lindstrom & Berhanu, 1999). Caldwell also, in his study involving effect of conflict on fertility, documented a fall followed by a rise in fertility for Russia, Spain and Germany in the context of major disruptions before the 1960's (Cadwell, 2004). Finlay, by use of cross sectional surveys compared fertility of residents of areas affected by earthquake, before and after the earthquake and documented that fertility rose after the disaster (Finley, 2009). A similar approach of study was also carried out by Hosseini and Abbasi who investigated the impact of the Bam erthquake in Iran and observed that Iran's fertility had declined in the year 2004 and then rose in 2005-2007 , which was far much later after the Bam earthquake which took place in 2004 (Hosseini & Abbasi, 2013). Demographers have adopted various demographic models to represent fertility in the form of mathematical functions relating some measurable fertility (Clogg & Eliason, 1988). Much has been done on modeling of fertility curves by researchers and many mathematical models have been proposed in order to describe the age specific fertility pattern. Many of these models have been shown to provide excellent fits to one-year age-specific fertility rate distributions of human populations (Hoem, 1981). In the year 2000, Gage made an improvement on Hoem's work in the year 2000. He tested the Gamma distribution, the Hadwiger function, and the Brass polynomial and extended to several non-human mammalian populations. He identified the optimum model for smoothing the distributions,in his work entitled Mathematical modeling of age specific fecundity distributions. He reported that all three models work well with a variety of mammalian data,the Brass polynomial being the simplest hence was the optimum choice (Gage, 2000). Gige had excluded the cubic spline, the Coale-Trussell function and the Beta and Gompertz functions because they did not fit contemporary human fecundity distributions well. He included the Brass polynomial despite the less accurate fits reported by Hoem because this procedure provided parameter values that are directly interpretable with respect to life history characteristics and because it has been used to smooth non-human fecundity distributions previously (Gage, 1995; Gage, 1998). In 2003, (Schmertmann, 2003), Schmertmann proposed an alternative model for representing age-specific fertility schedules. This is obtained by defining three index ages that describe the shape of the age-specific fertility using a piecewise quadratic Spline function. For all the modeling processes described above, interference to fertility has not been captured. Literature on modeling of fertility in the presence of interference is relatively scarce, hence the major motivation for this work. In this study, we hypothesize that interference may increase fertility rate hence cause a change in the fertility fertility pattern. This study statistically used R software to model fertility data of Rwanda 1992 and Rwanda 2000 so as to investigate the effect that the 1994 Genocide interference had on the Rwanda fertility distribution. It is our assumption that interference change fertility rate hence fertility pattern. Our study first fits probability distribution to interference free data set of Rwanda 1992 DHS), then, fits probability distribution to data set of Rwanda 2000 DHS that contain interference. We then model fertility rate and relate it to population growth. Since random factors affect all areas of demography and businesses striving to succeed in today's highly competitive environment, there is need for a tool to deal with uncertainty involved. Probability distributions would be a scientific tool to the demographers and business persons for use in dealing with uncertainty and making informed decisions in their businesses. The model of the random process developed would also protect demographers from potential time and money loss which can arise due to invalid model selection hence, helping the Government in planning for social amenities for its citizens.

2. Data and Methodology

The following data sets were used in our study. The Rwanda 1992 Demographic and Health survey data, The Rwanda 2000 Demographic and Health survey data and The Rwanda 2005 Demographic and Health survey data.

2.1 Modeling of fertility rate in the presence of interference

This section uses life table approach to determine net fertility rate (NFR) and links it to population growth. For comparison purposes, the chapter fits probability distribution to both interference free data and also to the data containing interference. Life table is a simple way of laying out the reproductive and mortality schedule of a population to aid in the measuring of population parameters. The two basic parameters are; age specific survival (mortality) rate and the age specific fertility rate, and from the two parameters, rate of population growth can be determined. A life table is a table of data on survivorship and fecundity of individuals within a population. Our study applies the cohort type of life table whose construction bases on to collection of data on a cohort or group of individuals all born in the same time period. The cohort life tables can then be used to determine the age specific fecundity and mortality rates, survivorship and basic reproduction rates.

2.2 The assumptions made in the Modeling

The population is closed (the net migration is Zero), the population is stationary, deaths and births are evenly spread through time. The table below shows stable age distribution - the proportion of individuals in each class is constant through time.

Table 1. Demography of a population (Variables of a life table)

Variable	Definition
x	Life stage or age group
a_x	Total number of individuals observed at each life stage or age group
l_o	The radix of the life table. It is a cohort of some arbitrary number of births on which a life table is based
l_x	Proportion of original number of individuals surviving to the next stage or age group (survivorship)
d_x	Proportion of original number of individuals dyeing during each stage or age group (mortality)
q_x	Mortality rate for each stage or group
m_x	Individual fecundity or mean reproductive output, for each stage

	or age group
$l_x m_x$	Number of offspring produced per original individual during each stage or group. (product of survival and fecundity)
R_0	Basic reproduction number
S_x	The likelihood of living (surviving) from birth to a given age, $Sx=l_x / l_0$

2.3 Relating fertility rate to population growth

Population variables depend on properties of the individuals that compose the population. The two basic parameters of a population are the individuals' likelihood of surviving and the individuals' likelihood to produce offspring. The parameters both depend on the individual's age.

2.4 Basic reproduction number, Also called, Net reproduction rate $R_{o}

In epidemiology, the transmissibility of an infection can be quantified by its basic reproductive number R_0, which is defined as the mean number of secondary infections produced by a single infection into a completely susceptible host population.

For many simple epidemic processes, this parameter determines a threshold: Whenever $R_0 > 1$, typical infective gives rise, on average, to more than one secondary infection, leading to an epidemic. In contrast, when $R_0 < 1$, infectious individual typically give rise, on average, to less than one secondary infection, and the prevalence of infection cannot increase ([Finley, 2009](#)). In the demography context, Basic reproduction number may represent the Net reproduction number or Net reproductive rate, R_0. Net reproductive rate R_0 is the average number of offspring produced by an individual in its lifetime, taking normal mortality into account. The word 'an individual' in this context means 'one female' and 'offspring' represent the 'female child'. This is because it is only females who carry the process of reproduction forwards in a population. It was an assumption in this study that only women who were between 15 years old and 49 years old were reproducing. $l_{x=}$ the proportion of original number of individuals surviving to the next stage or class (survivorship). By definition, $m_{x=}$ average number of offspring produced or mean reproductive output, for each stage or age group then $l_x m_x$= the number of offspring produced per original individual of age group x or (product of survival and fecundity). And, summing across all ages, this gives the average lifetime reproduction. Thus, $R_o = \sum l_x m_x$. If, $R_0 < 1$, individuals not fully replacing themselves, hence the population is shrinking. If, $R_0 = 1$, individual exactly replacing themselves, and the population size is stable. If, $R_0 > 1$, individuals more than replacing themselves, and the population is growing. R_0 is also called the replacement rate.

2.5 Basic fertility number F_0

m_x = half the number of offspring born to parent of age x. For each offspring produced, male and female parent each credited with 1/2 of an offspring produced. This is because, in sexual organism, each individual must produce two offspring for exact replacement. In practice, m_x is measured as female offspring per female of age x (m for maternity). This is simply because paternity is usually unknown, so numbers of offspring per male cannot be measured. It is therefore clear that, $R_0 = \frac{1}{2}F_0$. If, $R_0 > 1$, implies that $\frac{1}{2}F_0 > 1$. Thus, $F_0 > 2$, implies that population is increasing. If, $R_0 = 1$, implies that $\frac{1}{2}F_0 = 1$. Thus, $F_0 = 2$, implies that population is stable. If, $R_0 < 1$, implies that $\frac{1}{2}F_0 < 1$. Thus, $F_0 < 2$, implies that population is shrinking. It was an assumption in this study that presence of interference increases fertility rate of a population. Letting ρ be interference presence such that $\rho \geq 1$ and $F_0 > 2$, then $\rho F_0 > 2$ and population increases.

2.6 Determination of Generation time (T)

Generation time (T) is the time it takes for a new born baby to produce a baby. This time is referred to as the lasting interval of time. Regarding calculation of generation time of cohort, it takes the average length of time between the birth of an individual and the birth of one of its own offspring. Like any average it is the sum of all those lengths of time from all offspring, divided by the total number of offspring. Let R_0 be = the average number of offspring born to a female at age x then $R_0 = l_x m_x$. On weighing each offspring by the age of the mother, x, and then summing across all ages, we obtain the mother's age when each offspring was born, summed across all offspring born in her life. On dividing the value with R_0, we obtain weighted average (Generation time). $T = \frac{\sum X l_x m_x}{\sum l_x m_x}$ hence, $T = \frac{\sum X l_x m_x}{\frac{1}{2}F_0}$. The denominator is $\frac{1}{2}F_0$, thus, in a stable population, $F_0 = 2$, so the denominator has no effect on generation time. In a growing population $F_0 > 2$and T is decreased, because it takes less time for a cohort to • replace itself. In a shrinking population F_0 =<2 and T is increased, because it takes longer for a cohort to replace itself. R_o measures reproduction on the basis of individual lifetimes (offspring produced per individual per lifetime), and most models of population growth measure growth on the basis of births - deaths per unit time, where time does not have to be a generation (often years are the units). The most common measure of population growth is the intrinsic rate of increase, r. $r = \frac{\ln \frac{1}{2} F_0}{T}$. When: F_o = 2, r = 0, stable population; $F_0 < 2$, $r > 0$, shrinking; $F_0 > 2$, $r > 0$, growing; $r = b - d$ where b = births/unit time and d = deaths/unit time. So units of r are individuals produced per unit time.

2.7 Fitting probability distribution to data

Fitting Distribution is the procedure of selecting a statistical distribution that best fits to a data set generated by some random process. It is the task of finding a mathematical function which represents in a good way a statistical variable. It involves the choosing of a probability distribution modeling the random variable. Statistical data modeling is a field of statistical reasoning that seeks to fit models to data without knowing what the true model is or might be . One therefore needs to learn the model by a process called statistical model identification which requires judgment and expertise and generally needs an iterative process of distribution choice, parameter estimation and quality of fit assessment.

2.8 The distribution fitting process was guided by the following steps:

Step one: Model /function choice

Step two: Estimation of parameters

Step three: evaluation of quality of fit

Step four: Goodness of fit statistical test

All the above steps were performed using the 'fitdistrplus' package available in the R software.

3. Results and Discussion

Table 2. Rwanda number of children ever born descriptive statistics for the years 1992, 2000 and 2005.

Year	Min.	!st Qu.	Median	Mean	sd	3rd Qu.	Max
1992	1.000	4.000	6.000	5.885	2.705	8.000	15.000
2000	1.000	4.000	6.000	6.330	2.846	8.000	16.000
2005	1.000	4.000	6.000	5.876	2.680	8.000	17.000

It was observed from the table that the average number of children born in the year 1992 was 5.885. This rose to 6.330 in the year 2000. This was a drastic increase after the year 1994 of Genocide interference. The average number of children born later reduced to 5.876, most probably this was due to the fact that the population was by then undergoing recovery from interference.

Table 3. Parameter estimates for Gamma distribution fitted to Rwanda 1992 fertility data

Distribution	Parameter	Estimate	Std error	AIC value
Gamma	shape	4.7675508	0.03924480	128990.5
	rate	0.8102551	0.00703378	

The shape parameter is greater than one thus the gamma distribution assumes a mounded but skewed shape. The mean of the distribution is greater than the standard deviation, which is responsible for the shape just mentioned. The Rwanda 2000 fertility data also follow a Gamma distribution (a modified Gamma distribution) with the following parameters.

Table 4. Parameter estimates for Gamma distribution fitted to Rwanda 2000 fertility data

Distribution	Parameter	Estimate	Std error	AIC value
Gamma	shape	4.9727711	0.048840440	92923.67
	rate	0.7870042	0.008133496	

The shape parameter is greater than one thus the gamma distribution assumes a mounded but skewed shape. The mean of the distribution is greater than the standard deviation, which is responsible for the shape just mentioned.

3.1 Comparison between gamma distributions fitted for Rwanda 1992 data and the Rwanda 2000 fertility data.

The figure below show the summary of the gamma fitted distributions to Indonesia 2002 fertility data and also to Indonesia 2007 fertility data.

Figure 1: Gamma distribution function fitted to Rwanda 1992 fertility data and also to Rwanda 2000 fertility data.

The summary of the results from table 2 and table 3 were as follows;

Year	Shape Parameter	Rate Parameter	AIC value
1992	4.768	0.810	128990.5
2000	4.973	0.787	92923.67

The shape parameter (α), increased by 4.3 percent in the year 2000 than in the year 1992.

The rate parameter ($\frac{1}{\beta}$), decreased by 2.9 percent in the year 2000 than in the year 1992.

The scale parameter (β), increased by 2.9 percent in the year 2000 than in the year 1992.

Increase in the alpha parameter lowered the skewness of the distribution.

Increase in the scale parameter stretched the range of the Gamma distribution.

On drawing the two gamma distributions on the same axes, we clearly see the effect of Interference.

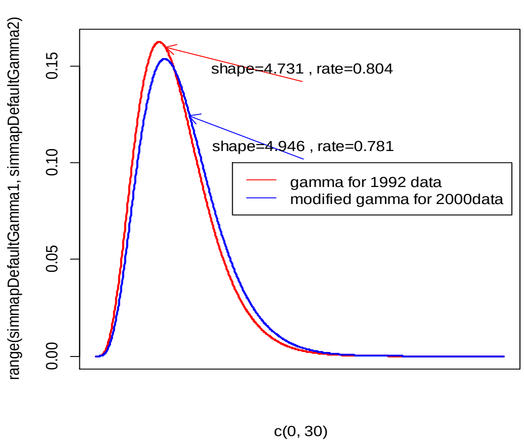

Figure 1: Gamma distribution for 1992 and 2000 fit to Rwanda data drawn on the same scale.

4. Conclusion

In this paper, we have given the Model of fertility rate in the presence of interference. These are predictive results that give growth rates of Rwanda. The results of this study are useful to demographers and businessmen in dealing with random factors that affect all areas of demography and businesses, striving to succeed in today's highly competitive environment.

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