# Mathematical Modeling and Analysis of Mathematics Anxiety Behavior on Mathematics Performance in Kenya

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*Authors' contributions*

This work was carried out in collaboration between both authors. Author NOM designed the study and wrote the first draft of the manuscript. Authors NOM and OKJ performed the mathematical analysis of the study and simulations. Author NOM managed literature searches. All authors read and approved the final manuscript.

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## Abstract

We propose a deterministic model that describes the dynamics of students who have the capability to perform well in mathematics examinations and engage in careers that demand its application and the negative influence of individuals with mathematics anxiety on the potential students. Our model is based on SIR classical infectious model classes with Susceptible (S) and Infected (I) taken as Math anxious students $(A_x)$ and Removed (R) adopted as achievers students $(A_a)$. The model is shown to be both epidemiologically and mathematically well posed. In particular, we prove that all solutions of the model are positive and bounded; and that every solution with initial conditions in $\Psi$ remains in the set $\Psi$ for all time. The existence of unique math anxious-free and endemic equilibrium points is proved and the basic reproduction number $\mathcal{R}_0$ computed using next generation matrix approach. A global stability of anxious-free and the endemic equilibria are performed using Lasselles invariance principle of Lyapunov functions. Sensitivity analysis shows that achievement rate of potential achievers $\delta$ and achievement rate of math anxious students $\gamma$ are the most sensitive parameters. This indicates that effort should be directed towards these parameters, by having well trained mathematics staff and the  best  printed  and  technological

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resources so as to control the spread of mathematics anxiety. Furthermore, scaling up the understanding level of mathematics algorithms, lowers the mathematics anxiety level and consequently, the spread of mathematics anxiety amongst students reduces. Lastly, some numerical simulations are performed to verify the theoretical analysis result using Matlab software.

# 1    Introduction

Competence in mathematics is paramount due to constant change in the global economy and workplace, use of mathematics for everyday decision making, the link between mathematics and other subjects and the intrinsic value of mathematical knowledge in every culture. Another factor that underscores the importance of mathematics is the fact that it is one of the two principal modes of engaging the world intellectually through words and numbers[1]. The greatest indicator of intellectual power entails the faculty of combining words with numbers. Mathematics is an extremely important subject and it is vital that students succeed in it.

Comprehension of mathematics by students is important since understanding of previous mathematics concepts enables students to learn and master new concepts[2]. The study by [3] accepts that, when students are given strong conceptual foundation through connectedness for effective learning of mathematical concepts, it improves interest and performance. This too is supported by [4]. Their study further confirmed that lack of comprehension by students may further impede and sent signs of frustration to students for higher studies in mathematics and courses that demand application of mathematics since there exists a relationship between previous and future mathematical achievements[4]. The studies by[5],[6] and [7] found out that students interest in mathematics increases when they understand the skills, how that skill is developed and how it is connected to needed mathematics competencies for performance. Mathematics is not a stagnant field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating new knowledge and understanding about the real world every day.

The mathematics teacher's ability to connect mathematics to real life problems is crucial to students interest development in mathematics. A number of studies carried out so far, reveal that there exists a strong relationship between teacher quality and student performance in mathematics. In Singapore for instance,[8] asserts that the problem of teaching mathematics needed qualified teachers/educators and recommended that the Ministry of Education equip mathematics teachers with the necessary skills through in-service courses and encouraging them to further their studies as much as possible. The study by [9] found the teachers ability to link Mathematics to other subject areasbeing the key to students interest development in mathematics. The study concluded that teachers' ability to connect mathematics to real life problems is very essential to their mathematics interest development process.

The disconnect between the real life problems and the learnt skills may lead to students struggling to understanding mathematics as echoed by [10]. Similar sentiments are echoed by [11] and that there is a correlation between the students mathematical struggle and students inability to solve problems. The difficulty in connecting mathematics skills to real life problems has been attributed to the traditional teaching methods adopted for instruction. This is seen to be contributing to the failure and bad performance of students [11].

More studies carried out recently to determine the relationship between teacher experience and students performance in mathematics found that teacher experience and competence were the prime

predictors of students performance in all subjects in secondary schools in Ondo state Nigeria [12]. The teachers are a key input and a force to reckon within school as observed in [13]. A similar observation is made by [14] about schools in Mississippi in USA, that scored better in mathematics when taught by teachers with more years of teaching. In South Africa, as pointed out by [15], few students take mathematics and those who do so, do not perform well because they are not motivated which ultimately may lead to mass failures.

Furthermore, it is observed that many schools do not offer mathematics and those that offer do not have adequate facilities for effective teaching and learning [15]. Similar sentiment are echoed by [16] that many teachers, students and parents have a negative attitude towards the teaching and learning of mathematics. The same views are supported by [17] and recommended that mathematics teachers and students be given incentives to raise their morale for better grades in mathematics. With the traditional undergraduate curriculum, students do not often regard themselves as active participants in mathematical exploration. Rather they are passive recipients of a body of knowledge, comprising definitions, rules and algorithms. Mathematics is usually taught as a right and wrong subject and as if getting the right answer were paramount. In contrast to most subjects, mathematics problems almost always have a right answer and the subject is often taught as if there was a right way to solve the problem and any other approaches would be wrong, even if students got the right answer. Mathematics is frequently taught with a rote learning behaviorist approach. That is;

- A problem set is introduced.

- A solution technique followed.

- Practice problems are repeated until mastery is achieved.

When learning, understanding the concepts should be paramount, but with a right/wrong approach to teaching mathematics, students are encouraged not to try, not to experiment, not to find algorithms that work for them, and not to take risks. This makes students become mathematics avoiders, thereafter limiting their future studies in the area of mathematics and are cut off from many occupations in the society. Further, this approach exposes learners to math anxiety. The biggest obstacle to a teacher is trying to teach students who experience math anxiety.

Math anxiety has been defined as an inconceivable dread of mathematics that can interfere with manipulating numbers and solving mathematical problems within a variety of everyday life and academic situations [18]. Mathematics anxiety is linked to poor mathematics performance on mathematics achievement tests, negative attitudes concerning mathematics and viewing the subject as being of little or no use to them outside schools as noted by [19]. In many countries, mathematics teachers are required only to obtain passing grades of 51 percent in mathematics examination, so that a mathematics student who has failed to understand 49 percent of the math syllabus throughout his/her education period can, and often does, become a mathematics teacher. His/her fears and lack of understanding is then passed naturally to his/her students.

The objective of this work is to describe the transmission process of math anxiety amongst students, which can be defined generally as follows: when a reasonable number of mathematics anxious students or math teachers are introduced into potential subgroup, math anxiety is passed on to other students through its modes of transmission, thus spreading in the student population. Mathematics anxiety is assumed to be a nonstandard epidemic process that rarely emerges out of nothing but is usually related to some already math anxious environment which may affect potential students. Thus, in this study we will present a deterministic mathematical model for the spread of math anxiety in the spirit of epidemiology which describes the dynamical behavior of math anxiety, as a disease by involving well trained teachers and use of excellent printed resources as intervention. Three sub-classes were formulated based on the level of mathematics anxiety and nonlinear first order ordinary differential equations which govern the dynamics of math anxiety deduced. The math

anxiety reproduction number was determined using the Next-generation matrix. Model analysis was done and analytical results obtained. Simulation was carried out to ascertain analytical results.

# 2    Model Formulation

In this study, we consider a mathematical model $(P, A_x, A_a)$ that describes the dynamics of students who have the potential to achieve good grades in the final examination, and the negative influence due to math anxiety on the potential achievers. We divide the total students population denoted by N into three compartments: The potential achievers sub-population P are susceptible students without math anxiety and they are capable of achieving good grades in mathematics. This class of potential achievers is increased by the recruitment of students into the compartment P at a rate $\Lambda$ and decreased when potential achievers, who are math anxious free, moves to achievers class at a rate $\delta$. It is assumed that potential achievers can acquire math anxiety via effective contact with students/teachers who are math anxious (math anxious students are students with false assumption that mathematical aptitude is innate and that only certain students will ever be able to succeed in mathematics) in compartment $A_x$ at a rate $\beta$. An individual with math anxiety does not necessarily lack ability in mathematics, rather, they cannot achieve their full potential due to the interfering symptoms of their math anxiety. Mathematics anxiety transmission probability per contact is represented by $\rho$. The parameter $\tau$ is assumed to be $0 \leq \tau < 1$, this estimates how well an individual understands mathematics algorithms, $\tau = 0$ if an individual understands nothing and highly likely to be affected with math anxiety and vice versa. Thus $\beta = \rho(1 - \tau)$ is the effective math anxiety contact rate. Students with math anxiety are increased when the potential achievers interact with math anxious students at a rate $\beta$. The parameter $\gamma$ measures the rate at which mathematically anxious students may be guided positively by qualified teachers, parents or even achiever students and join achievers subclass. The achievers class $A_a$ are positive about mathematics and they register good grades, this compartment is increased at a rate $\delta$ and $\gamma$ when potential achievers and mathematically anxious students join the achievers compartment respectively. All three compartments are decreased by natural death rate $\mu$. P(t), $A_x(t)$, and $A_a(t)$ are variables that represent numbers of the individuals in the three compartments at time t in years. This study assumes the following: homogeneous mixing of the students, that it is not possible for individuals in subclass $A_a$ to recover to subclass $A_x$, the net effect of dropouts and parents influence is not significant, net conversion from $A_x$ to $A_a$ similarly from $P$ to $A_a$ is positive and it is not possible for individuals in $A_a$ to recover to P. The total population size at time t is denoted by N(t) with $N(t) = P(t) + A_x(t) + A_a(t)$.

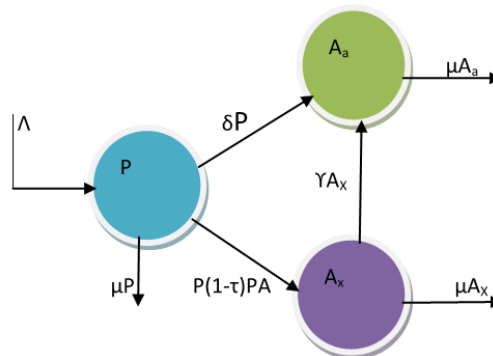Graphical representation of the proposed model is given in figure 1 below.



**Fig. 1. Flow chart diagram of the proposed model**

The dynamics of the proposed model is governed by the following nonlinear system of differential equations:

$$\begin{cases} \frac{dP}{dt} = \Lambda - \beta P(t)A_x(t) - \delta P(t) - \mu P(t), \\ \frac{dA_x}{dt} = \beta P(t)A_x(t) - \gamma A_x(t) - \mu A_x(t) \\ \frac{dA_a}{dt} = \gamma A_x(t) + \delta P(t) - \mu A_a(t), \end{cases} \qquad (1)$$

Throughout this paper, we assume that the initial conditions of system (1) are non-negative: Where $P(0) > 0$, $A_x(0) \geq 0$, and $A_a(0) > 0$, and $\beta = \rho(1-\tau)$ is the effective math anxiety contact rate.

# 3 Basic Properties of the Model

## 3.1 Positivity of the model's solutions

Since the model suggests human population, we need to show that all the state variables remain non-negative for all times.

**Lemma 1**. Let $\Psi = \left\{ (P(t), A_x(t), A_a(t)) \in \mathcal{R}_+^3 : P(0) \geq 0, A_x(0) \geq 0, A_a(0) \geq 0 \right\}$ then the solutions of P(t) , $A_x(t)$ , $A_a(t)$ of the system (1) are positive for all $t \geq 0$.

*Proof*. From the first equation of system (1), $\frac{dP}{dt} = \Lambda - \beta P(t)A_x(t) - (\delta + \mu)P(t)$, it follows that $\frac{dP}{dt} \geq -(\delta + \mu)P(t)$. On integration of $\frac{dP}{dt} \geq -(\delta + \mu)P(t)$, we obtain; $\frac{d}{dt}[P(t)e^{\{-\int_0^t (\delta+\mu)dp\}}] \geq 0$. Clearly, $P(t)e^{\{-\int_0^t (\delta+\mu)dt\}}$ is non-negative function of time (t), thus P(t) stays positive. Similarly, we can show that; $A_x(t) \geq A_x(0)e^{-(\gamma+\mu)t} \geq 0$ and $A_a(t) \geq A_a(0)e^{-\mu t} \geq 0$. Therefore all solutions of the system of equations (1) are positive for all $t \geq 0$ in the region $\Psi$. $\qquad \square$

## 3.2 Invariant region

**Lemma 2**. The system of equations (1) has solutions which are contained, in the feasible region $\Psi$.

*Proof*. If we let $\Psi = (P(t), A_x(t), A_a(t)) \in \mathcal{R}_+^3$ be any solution of the system of equations 1 with non-negative initial conditions then adding the equations of the system (1), we have $\left\{ \frac{dN}{dt} = \Lambda - \mu(P(t) + A_x(t) + A_a(t)) \right\}$, that is $\frac{dN}{dt} = \Lambda - \mu N(t)$. Applying the integrating factor method, $N(t) = \frac{\Lambda}{\mu} + Ce^{-\mu t}$, at $t = 0$, $N(0) = N_0$ and $C = N_0 - \frac{\Lambda}{\mu}$ so that $N(t) = \frac{\Lambda}{\mu} + \{N_0 - \frac{\Lambda}{\mu}\}e^{-\mu t}$. Whenever $N > \frac{\Lambda}{\mu}$, then $\frac{dN}{dt} < 0$, implying $\frac{dN}{dt}$ is bounded by $\Lambda - \mu N$. Thus, a standard comparison theorem [20] can be used to show that $N(t) \leq N(0)e^{-\mu t} + \frac{\Lambda}{\mu}(1 - e^{-\mu t})$, in particular $N(t) \leq \frac{\Lambda}{\mu}$ if $N(0) \leq \frac{\Lambda}{\mu}$. Thus, $\Psi$ is positively-invariant meaning all solutions in $\Psi$ remain in $\Psi$. Furthermore, if $N(t) > \frac{\Lambda}{\mu}$, then either the solution enters $\Psi$ in finite time or N(t) approaches $\frac{\Lambda}{\mu}$ and the infected variable $A_x$ approaches zero. Therefore, $\Psi$ is attracting, in that, all solutions in $\mathcal{R}_+^3$ eventually enter $\Psi$. $\qquad \square$

Thus, from lemma 1 and 2 the model is well-poised epidemiologically and mathematically [21] and hence, it is sufficient to study the dynamics of the system (1) in $\Psi$.

# 4 Stability Analysis of the Model Fixed Points

In this section, we will study the stability behavior of system (1) at math anxious-free equilibrium point and endemic equilibrium point.

## 4.1 Math Anxious Free Equilibrium $(A_x.F.E)$ point

The math anxious-free equilibrium point is the state in which the population is free of math anxious influence so that, we have only Potential achiever and the Achievers individuals. Thus, the model has math anxious-free equilibrium, obtained by setting the right-hand side of the system of equations (1) to zero and $A_x(t) = 0$ , gives:

$$E^0 = (P(t), A_x(t), A_a(t)) = \left\{ \frac{\Lambda}{(\delta + \mu)}, 0, \frac{\delta\Lambda}{\mu(\delta + \mu)} \right\}. \tag{2}$$

## 4.2 The basic reproduction number $\mathcal{R}_0$

The reproduction number $\mathcal{R}_0$ is the most valuable concept that mathematical thinking has brought to epidemic theory, where control strategies are employed [22]. The reproduction number represents the total number of new math anxious cases recorded, throughout the contact period when a math anxious individual is introduced into a purely potential achievers population. We obtained the basic reproduction number, $\mathcal{R}_0$ of the system of equations (1) which is the spectral radius $\rho$ of the next generation matrix, M, that is $\mathcal{R}_0 = \rho M$ , where $M = FV^{-1}$ . The matrices of F (for the new infection terms) and V (of the transition terms) are obtained from the math anxious compartment (i.e infected compartment $A_x(t)$ ) at anxious-free equilibrium $E^0$ .

**Lemma 3.** The basic reproduction number of the mathematical model (1) is given by;

$$\mathcal{R}_0 = \frac{\beta\Lambda}{(\delta + \mu)(\gamma + \mu)} \tag{3}$$

*Proof.*

$$\mathcal{F} = \left\{ \begin{array}{c} \mathcal{F}_{A_x} \\ \mathcal{F}_{A_a} \end{array} \right\} = \left\{ \begin{array}{c} \beta P A_x \\ 0 \end{array} \right\}, and, \mathcal{V} = \left\{ \begin{array}{c} \mathcal{V}_{A_x} \\ \mathcal{V}_{A_a} \end{array} \right\} = \left\{ \begin{array}{c} (\gamma + \mu) A_x \\ -\gamma A_x - \delta P + \mu A_a \end{array} \right\} \tag{4}$$

Matrices F and V are obtained by taking partially derivatives of $A_x$ and $A_a$ with time t as follows;

$$F = \left\{ \begin{array}{cc} \frac{\partial \mathcal{F}_{A_x}}{\partial A_x} & \frac{\partial \mathcal{F}_{A_x}}{\partial A_a} \\ \frac{\partial \mathcal{F}_{A_a}}{\partial A_x} & \frac{\partial \mathcal{F}_{A_a}}{\partial A_a} \end{array} \right\} And, V = \left\{ \begin{array}{cc} \frac{\partial \mathcal{V}_{A_x}}{\partial A_x} & \frac{\partial \mathcal{V}_{A_x}}{\partial A_a} \\ \frac{\partial \mathcal{V}_{A_a}}{\partial A_x} & \frac{\partial \mathcal{V}_{A_a}}{\partial A_a} \end{array} \right\} \tag{5}$$

The Jacobian matrices of F and V at anxious free equilibrium point are given by;

$$F = \left\{ \begin{array}{cc} \frac{\beta\Lambda}{\delta + \mu} & 0 \\ 0 & 0 \end{array} \right\} And, V = \left\{ \begin{array}{cc} \gamma + \mu & 0 \\ -\gamma & \mu \end{array} \right\} \tag{6}$$

The inverse of matrix $V$ is worked out as;

$$V^{-1} = \frac{1}{\mu(\gamma + \mu)} \left\{ \begin{array}{cc} \mu & 0 \\ \gamma & \gamma + \mu \end{array} \right\} \tag{7}$$

The characteristic polynomial $FV^{-1} - \lambda I_2 = 0$ become;

$$|(FV^{-1}) - \lambda I_2| = \left| \begin{array}{cc} \frac{\beta\Lambda}{(\gamma + \mu)(\gamma + \mu)} - \lambda_1 & 0 \\ 0 & 0 - \lambda_2 \end{array} \right| = 0 \tag{8}$$

Evaluating the roots of the characteristic polynomial (8), we have;

$$\lambda_1 = \frac{\beta\Lambda}{(\gamma + \mu)(\gamma + \mu)}$$
$$\lambda_2 = 0 \tag{9}$$

Since the matrix (8) has only one non-zero eigenvalue, the spectral radius of system of equations (1) given as $\mathcal{R}_0 = \rho \mathcal{F} \mathcal{V}^{-1}$ is thus obtained as;

$$\mathcal{R}_0 = \rho \mathcal{F} \mathcal{V}^{-1} = \frac{\beta \Lambda}{(\delta + \mu)(\gamma + \mu)} \tag{10}$$

This concludes the proof. $\qquad \square$

## 4.3 Local Stability of the Math anxious Free Equilibrium ($A_x.F.E$) point

**Theorem 1.**The anxious-free equilibrium, $E^0$ of the model is locally asymptotically stable (LAS) if $\mathcal{R}_0 < 1$ on $\Psi$.

*Proof.* The characteristic polynomial of the corresponding linearized system of the governing system of equations (1) is given as $p(\lambda) = det((F - V) - \lambda I_3))$. From the system of equations (1), the Jacobian matrix evaluated at $E^0 = (P^0, 0, A_a^0)$ the anxious free equilibrium is given by;

$$|J(E_o) - \lambda I| = \begin{vmatrix} -\{\delta + \mu\} - \lambda_1 & \frac{-\beta \Lambda}{\delta + \mu} & 0 \\ 0 & \frac{\beta \Lambda}{\delta + \mu} - \{\gamma + \mu\} - \lambda_2 & 0 \\ \delta & \gamma & -\mu - \lambda_3 \end{vmatrix} = 0 \tag{11}$$

Evaluating the roots of this characteristic polynomial, we have;

$$\begin{cases} \lambda_1 = -\{\delta + \mu\} < 0 \\ \lambda_2 = \frac{\beta \Lambda}{\delta + \mu} - \{\gamma + \mu\} \\ \lambda_3 = -\mu < 0. \end{cases} \tag{12}$$

Now, for $\lambda_2$ to be negative, we must have;

$$\frac{\beta \Lambda}{\delta + \mu} - \{\gamma + \mu\} < 0 \tag{13}$$

Simplifying equation (13) we obtain;

$$\frac{\beta \Lambda}{\{\gamma + \mu\}\{\delta + \mu\}} = \mathcal{R}_0 < 1 \tag{14}$$

Thus, $\lambda_2 < 0$ if and only if $\mathcal{R}_0 < 1$. Using the Routh-Hurwitz criterion [23], it can be seen that all the eigenvalues of the characteristic polynomial (11) have negative real part if and only if $\mathcal{R}_0 < 1$. Hence $E^0$ is LAS on $\Psi$ provided the inequality $\mathcal{R}_0 < 1$ is satisfied. This completes the proof. $\quad \square$

The epidemiological implication of the local stability is that math anxiety influence can be under control (bearable) in the population when $\mathcal{R}_0 < 1$ if the initial sizes of the sub-populations of the model are in the basin of attraction of $(E^0)$. In order to ensure that math anxiety influence is independent of the initial sizes of the sub-populations of the model, it is necessary to show that $E^0$ is globally-asymptotically stable (GAS) [24].

## 4.4 Global stability of math anxious free equilibrium ($A_x.F.E$) point

**Theorem 2.**If $\mathcal{R}_0 \leq 1$, then the anxious-free equilibrium $E^0$ is globally asymptotically stable on $\Psi$.

*Proof.* To achieve this, we consider the following lyapunov function:

$$M(t) = \int_{P^0}^{P} \{1 - \frac{P^0}{y}\} dy + A_x(t) \tag{15}$$

The derivative of M along the solution of model (1) is;

$$M(t)' = \{1 - \frac{p^0}{p(t)}\} \frac{dP}{dt} + \frac{dA_x}{dt} \tag{16}$$

Substituting in for $\frac{dP}{dt}$ and $\frac{dA_x}{dt}$ we have;

$$\begin{cases} M(t)' = \{\delta + \mu\}\{1 - \frac{P^o}{P}\}\{P^o - P\} - \beta P A_x\{1 - \frac{P^o}{P}\} \\ + \beta P A_x - \{\gamma + \mu\}A_x \end{cases} \tag{17}$$

With slight manipulation of equation (17) and using $P^o = \frac{\Lambda}{\delta + \mu} \Leftrightarrow \Lambda = \{\delta + \mu\}P^o$ we have;

$$\begin{cases} M(t)' = \{\delta + \mu\}\{1 - \frac{P^o}{P}\}\{P^o - P\} - \beta P A_x\{1 - \frac{P^o}{P}\} \\ + \beta P A_x - \{\gamma + \mu\}A_x \end{cases} \tag{18}$$

Expanding and simplifying equation (18);

$$M(t)' = -\{\delta + \mu\}\frac{\{P - P^o\}^2}{P} + \beta P^o A_x - \{\gamma + \mu\} \tag{19}$$

Introducing $\mathcal{R}_0$ {from equation 3} into equation (19) gives;

$$M(t)' = -\{\delta + \mu\}\frac{\{P - P^o\}^2}{P} + \{\gamma + \mu\}\{\mathcal{R}_0 - 1\}A_x \tag{20}$$

Thus, $M(t) \leq 0$ if $\mathcal{R}_0 \leq 1$ with $M(t)' = 0$ if and only if $\mathcal{R}_0 = 1$, $P = P_o$ and $A_x = 0$. Substituting $A_x = 0$ in the system of equations (1) shows that $P(t) \to \frac{\Lambda}{(\delta + \mu)}$ as $t \to \infty$ and $A_a(t) \to \frac{\delta \Lambda}{\mu(\delta + \mu)}$ as $t \to \infty$ (see also figure 3 and 4). Further, the largest compact invariant set in $\{(P(t), A_x(t), A_a(t)) \in \Psi : M(t)' = 0\}$ is the singleton $E_0$. It follows from [25], that every solution to system of equations (1) with initial conditions in $\Psi$ converges to the $(AFE)_{E^0}$ as $t \to \infty$ whenever $\mathcal{R}_0 \leq 1$, that is $(P(t), A_x(t), A_a(t)) \to \{\frac{\Lambda}{(\delta + \mu)}, 0, \frac{\delta \Lambda}{\mu(\delta + \mu)}\}$ as $t \to \infty$. This completes the proof. $\square$

The global stability of anxious free equilibrium (GAS) shows that math anxiety will be under control regardless of the initial profile of the anxious students population if $\mathcal{R}_0$ can be maintained at a unit or less than a unit (see figures 3 and 4 for $\mathcal{R}_0 < 1$ and $\mathcal{R}_0 = 1$, $(P(t), A_x(t), A_a(t)) \to \{\frac{\Lambda}{(\delta + \mu)}, 0, \frac{\delta \Lambda}{\mu(\delta + \mu)}\}$) as $t \to \infty$.

## 4.5   Existence of math anxious-endemic equilibrium

At math anxious-endemic equilibrium ($A_x.E.E$) we have persistence of math anxiety infection in the population. Therefore, we need to find the positive endemic equilibrium of the system of equations (1), denoted by:

$$E^* = \{P(t)^*, A_x(t)^*, A_a(t)^*\} \tag{21}$$

**Lemma 4.** The system of equations (1) has a unique endemic (positive) equilibrium whenever $\mathcal{R}_0 > 1$, and no positive equilibrium otherwise.

*Proof.* The equations in the system (1) are solved in terms of the associated force of recruitment at steady-state, given by:

$$\vartheta = \beta P(t)^* A_x(t)^* \tag{22}$$

Solving the system of equations of the model (1) at steady state and factoring in $\mathcal{R}_0$ gives:

$$\begin{cases} P^* = \frac{\Lambda}{\{\delta+\mu\}\mathcal{R}_0} \\ A_x^* = \frac{\{\delta+\mu\}}{\beta}\{\mathcal{R}_0 - 1\} \\ A_a^* = \frac{\gamma\{\delta+\mu\}\{\mathcal{R}_0-1\}+\delta\{\gamma+\mu\}}{\mu\beta}. \end{cases} \tag{23}$$

$E^* = (P^*, A_x^*, A_a^*)$ is equilibrium point corresponds to the case where math anxiety infection invade the student population. Using the first and second equations of system (23) in (22) and simplifying gives:

$$\vartheta = \Lambda\{1 - \frac{1}{\mathcal{R}_0}\} \tag{24}$$

Since all model parameters are assumed non-negative with $\mu > 0$, it follows that $\vartheta > 1$ is non-negative whenever $\mathcal{R}_0 > 1$. Noting that $\mathcal{R}_0 < 1$ implies that the force of recruitment $\vartheta$ at steady state is negative (which is biologically meaningless) hence, the model has no positive equilibrium in this case. Thus, the lemma is established. This completes the proof. □

## 4.6 Local stability of math anxious-endemic equilibrium

A similar approach as that used in local stability of anxious-free equilibrium is applied here. The Jacobian stability approach to prove the stability of the anxious endemic equilibrium state is used.

**Theorem 3.** The positive endemic equilibrium state of the system (1) is locally asymptotically stable (LAS) when $\mathcal{R}_0 > 1$ on $\Psi$.

*Proof.* The Jacobian matrix of system (1) at $E^*$ is given by;

$$J_{E_o} = \begin{Bmatrix} -\{\beta A_x^* + \{\delta+\mu\}\} & -\beta P^* & 0 \\ \beta A_x^* & \beta P^* - \{\gamma+\mu\} & 0 \\ \delta & \gamma & -\mu \end{Bmatrix} \tag{25}$$

Applying elementary row operation, matrix (25) reduces to;

$$J_{E_o} = \begin{Bmatrix} -\{\beta A_x^* + \{\delta+\mu\}\} & -\beta P^* & 0 \\ 0 & \beta P^* - \{\gamma+\mu\} - \frac{\beta^2 A^* P^*}{\beta A_x^* + \{\delta+\mu\}} & 0 \\ 0 & 0 & -\mu \end{Bmatrix} \tag{26}$$

The characteristic polynomial of the upper triangular matrix (26) is given as;

$$J_{E_o} = \begin{vmatrix} -\{\beta A_x^* + \{\delta+\mu\}\} - \lambda_1 & -\beta P^* & 0 \\ 0 & \beta P^* - \{\gamma+\mu\} - \frac{\beta^2 A_x^* P^*}{\beta A_x^* + \{\delta+\mu\}} - \lambda_2 & 0 \\ 0 & 0 & -\mu - \lambda_3 \end{vmatrix} = 0 \tag{27}$$

Thus, the roots of this characteristic polynomial are;

$$\begin{cases} \lambda_1 = -\{\beta A_x^* + \{\delta+\mu\}\} < 0 \\ \lambda_2 = \beta P^* - \{\gamma+\mu\} - \frac{\beta^2 A_x^* P^*}{\beta A_x^* + \{\delta+\mu\}} \\ \lambda_3 = -\mu < 0. \end{cases} \tag{28}$$

Now, for $\lambda_3$ to be negative, we must have;

$$\frac{\beta P^*\{\delta + \mu\}}{\beta A_x^* + \{\delta + \mu\}} < \{\gamma + \mu\} \tag{29}$$

A slight manipulation of inequality (29) and factoring in $P^*$ and $A_x^*$ from equation (23) gives;

$$\{\mathcal{R}_0 - 1\} > 0 \tag{30}$$

Thus, $\lambda_3 < 0$ if $\mathcal{R}_0 > 1$ by the Routh-Hurwitz criterion [23], it can be seen that all the eigenvalues of the characteristic polynomial (27) have negative real part if and only if $\mathcal{R}_0 > 1$. Thus, the theorem is established. This completes the proof. $\qquad \square$

## 4.7 Global stability of math anxious-endemic equilibrium

**Theorem 4.** If $\mathcal{R}_0 > 1$, then the anxious-endemic equilibrium, $E^*$ of system (1) is globally asymptotically stable (GAS) on $\Psi$. At endemic equilibrium, system of equations (1) can be expressed as;

$$\begin{cases} \Lambda = \beta A_x^* P^* + \{\delta + \mu\}P^* \\ \gamma + \mu = \beta P^* \\ \mu = \frac{\gamma A_x^* + \delta P^*}{A_a^*} \end{cases} \tag{31}$$

*Proof.* Consider the Lyapunov function $U(t) : \psi \longrightarrow R$ given by;

$$\begin{cases} U(P, A_x) = \{P - P^* - P^* ln(\frac{P}{P^*})\} + \{A_x - A_x^* - A_x^* ln(\frac{A_x}{A_x^*})A_x\} \\ + A_a^*\{A_a - A_a^* - A_a^* ln(\frac{A_a}{A_a^*})A_a\} + \frac{1}{2\mu}\{(P - P^*) + (A_x - A_x^*) \\ + (A_a - A_a^*)\}^2. \end{cases} \tag{32}$$

The time derivatives along the solutions of the model equations (1) is given by;

$$\begin{cases} U'(P, A_x) = (1 - \frac{P^*}{P})P' + (1 - \frac{A_x^*}{A_x})A_x' + A_a^*(1 - \frac{A_a^*}{A_a})A_a' \\ + \frac{1}{\mu}\{(P - P^*) + (A_x - A_x^*) + (A_a - A_a^*)\}\{P' + A_x' + A_a'\} \end{cases} \tag{33}$$

Factoring in $P(t)'$, $A_x(t)'$ and $A_a(t)'$ in equation (33) and using equation (31) we have;

$$\begin{cases} U' = \{1 - \frac{P^*}{P}\}\{\beta P^* A_x^* + (\delta + \mu)P^* - \beta P A_x - (\delta + \mu)P\} \\ + \{1 - \frac{A_x^*}{A_x}\}\{\beta P A_x - \beta A_x P^*\} + A_a\{1 - \frac{A_a^*}{A_a}\} \\ \{\gamma A_x + \delta P - \frac{\gamma A_a A_x^* + \delta A_a P^*}{A_a^*}\} + \frac{1}{\mu}[\{(P - P^*) + (A_x - A_x^*) \\ + (A_a - A_a^*)\}\{\mu(P^* + A_x^* + A_a^*) - \mu(P + A_x + A_a)\}] \end{cases} \tag{34}$$

Simplifying (34) we have;

$$\begin{cases} U' = -(\delta + \mu)\frac{(P - P^*)^2}{p} + \beta P^* A_x^*\{(1 - \frac{\beta P A_x}{\beta P^* A_x^*})(1 - \frac{P^*}{P}) \\ + (\frac{\beta P A_x}{\beta P^* A_x^*} - \frac{A_x}{A_x^*})(1 - \frac{A_x^*}{A_x})\} + (1 - \frac{A_a^*}{A_a})\{\gamma(A_x A_a^* - A_x^* A_a) \\ + \delta(P A_a^* - P^* A_a)\} - \{(P - P^*) + (A_x - A_x^*) \\ + (A_a - A_a^*)\}\{(P - P^*) + (A_x - A_x^*) + (A_a - A_a^*)\} \end{cases} \tag{35}$$

Letting $x = \frac{P(t)}{P(t)^*}$, $y = \frac{A_x(t)}{A_x(t)^*}$, $z = \frac{A_a(t)}{A_a(t)^*}$ and $\delta = \gamma = 1$ equation (35) reduces to

$$\begin{cases} U' = -\mu\frac{(P - P^*)^2}{P} - (A_a - A_a^*)^2 + \beta P^* A_x^*\{(1 - xy)(1 - \frac{1}{x}) \\ + (xy - y)(1 - \frac{1}{y})\} - \{(P - P^*) + (A_x - A_x^*)\}\{(P - P^*) \\ + (A_x - A_x^*) + (A_a - A_a^*)\} + \frac{(A_a - A_a^*)}{A_a}\{(P A_a^* - P^* A_a)\} \\ - (A_a - A_a^*)(P - P^*) + \frac{(A_a - A_a^*)}{A_a}\{(A_x A_a^* - A_x^* A_a)\} \\ - (A_a - A_a^*)(A_x - A_x^*) \end{cases} \tag{36}$$

A further simplification of (36) gives;

$$
\begin{cases}
U' = -\{\mu \frac{(P-P^*)^2}{P} + (A_a - A_a^*)^2 + \{(P - P^*) + (A_x - A_x^*)\} \\
\{(P - P^*) + (A_x - A_x^*) + (A_a - A_a^*)\}\} + \beta P^* A_x^* \{(1 - xy) \\
(1 - \frac{1}{x}) + (xy - y)(1 - \frac{1}{y})\} + \{2A_a^*(P + A_x) - A_a(P + A_x) \\
- \frac{A_a^* A_a^*}{A_a}(P + A_x)\}
\end{cases}
\tag{37}
$$

equation (37) reduces to;

$$
\begin{cases}
U' = -\{\mu \frac{(P-P^*)^2}{P} + (A_a - A_a^*)^2 + \{(P - P^*) + (A_x - A_x^*)\} \\
\{(P - P^*) + (A_x - A_x^*) + (A_a - A_a^*)\}\} + \beta P^* A_x^* t_1 + t_2
\end{cases}
\tag{38}
$$

Where $t_1 = \{(1-xy)(1-\frac{1}{x})+(xy-y)(1-\frac{1}{y})\}$, $t_2 = \{2A_a^*(P+A_x)-A_a(P+A_x)-\frac{A_a^* A_a^*}{A_a}(P+A_x)\}$.
Starting with $t_1$ and letting $x = y$ we have;

$$
\begin{cases}
t_1 = 2 - y - \frac{1}{y}, i.e \\
t_1 = 2 - \frac{A_x}{A_x^*} - \frac{A_x^*}{A_x}
\end{cases}
\tag{39}
$$

Similarly simplifying $t_2$ we have;

$$
\begin{cases}
t_2 = (P + A_x)A_a^* \{\frac{A_a - A_a^*}{A_a} + \frac{A_a^* - A_a}{A_a^*}\} i.e \\
t_2 = (P + A_x)A_a^* \{2 - \frac{A_a^*}{A_a} - \frac{A_a}{A_a^*}\}
\end{cases}
\tag{40}
$$

Thus, Equation (37), becomes;

$$
\begin{cases}
U' = -\{\mu \frac{(P-P^*)^2}{P} + (A_a - A_a^*)^2 + \{(P - P^*) + (A_x - A_x^*)\} \\
\{(P - P^*) + (A_x - A_x^*) + (A_a - A_a^*)\}\} + \beta P^* A_x^* \{2 - \frac{A_x}{A_x^*} - \frac{A_x^*}{A_x}\} \\
+(P + A_x)A_a^* \{2 - \frac{A_a^*}{A_a} - \frac{A_a}{A_a^*}\}
\end{cases}
\tag{41}
$$

The arithmetic-geometric mean inequality holds for the following inequalities;

$$
\{2 - \frac{A_x}{A_x^*} - \frac{A_x^*}{A_x}\}, \{2 - \frac{A_a^*}{A_a} - \frac{A_a}{A_a^*}\} \le 0
\tag{42}
$$

Thus, if $\frac{A_x}{A_x^*} = \frac{A_a}{A_a^*} = 1$ then $U'(P, A_x, A_a) \le 0$, further more $\{U'(P, A_x, A_a) = 0, P \Longleftrightarrow P^*\}$ and therefore the largest invariant set of system (1) on the set $\Psi$ is the endemic equilibrium point $E^*$. Hence, by [25], the free equilibrium point $E^*$ is globally asymptotically stable in $\Psi$, and hence, the proof is complete. □

# 5 Numerical Results

We estimated parameters, numerical sensitivity analysis and numerical simulations in this section.

## 5.1 Parameter estimation

The assumed total population and recruitment rate is related by $\Lambda = \mu N$. According to [26], life expectancy in Kenya was estimated as 63.52 years. This study therefore, assumes constant death rate $(\mu)$ to be the reciprocal of the life expectancy. Parameters value estimation is presented in the Tables 2 and 3. If $\mathcal{R}_0 < 1$, then on average, an individual with math anxiety produces less than one newly math anxious individual over the course of recruitment period.

In this case, the math anxiety influence may die out in the long run. Conversely, if $\mathcal{R}_0 > 1$ , each individual with math anxiety produces, on average more than one new math anxious individual. In this case, mathematics anxiety influence will spread in the population. A large value of $\mathcal{R}_0 > 1$ may indicate the possibility of a major problem.

**Table 1. Description of parameters**

| Parameters | Description |
|---|---|
| $\Lambda$ | Recruitment rate |
| $\delta$ | Achievement rate of potential achievers |
| $\mu$ | Natural death rate |
| $\gamma$ | Achievement rate of math anxious students |
| $\mathcal{R}_0$ | Reproduction number |
| $t_f$ | Number of days per year |
| $\tau$ | Understanding level of mathematics algorithms |
| $\rho$ | Math anxiety influence transmission |

**Table 2. Baseline values for parameters of the system (1) $\mathcal{R}_0 < 1$**

| Parameter | Value | Reference |
|---|---|---|
| $\Lambda$ | 11,220 | Estimated |
| $\delta$ | 0.0108 | Estimated |
| $\mu$ | 0.016 | [27] |
| $\gamma$ | 0.008 | Estimated |
| $\mathcal{R}_0$ | 0.732649253 | Calculated |
| $t_f$ | 365 day | a year |
| $\tau$ | 0.6 | Assumed |
| $\rho$ | $1.05 \times 10^{-7}$ | Estimated |

**Table 3. Baseline values for parameters of the system (1) $\mathcal{R}_0 > 1$**

| Parameter | Value | Reference |
|---|---|---|
| $\Lambda$ | 11,220 | Estimated |
| $\delta$ | 0.0108 | Estimated |
| $\mu$ | 0.016 | [27] |
| $\gamma$ | 0.008 | Estimated |
| $\mathcal{R}_0$ | 1.465298507 | Calculated |
| $t_f$ | 365 day | a year |
| $\tau$ | 0.2 | Assumed |
| $\rho$ | $1.05 \times 10^{-7}$ | Estimated |

Table 2 indicate parameter estimate for system of equations (1), the anxious-free equilibrium is found to be $(418657, 0, 282593)$ and $\mathcal{R}_0 = 0.732649253 < 1$. In this case, according to theorem (2), the anxious-free equilibrium $E^0$ of system of equations (1) is globally asymptotically stable on $\Psi$ (see Fig. 3).

Similarly we have the anxious endemic equilibrium $E^*$ as $(285714, 148453, 267083)$ and $\mathcal{R}_0 = 1.465298507 > 1$. In this case, according to theorem (4), the anxious endemic equilibrium $E^*$ of system (1) is globally asymptotically stable on $\Psi$ (see Fig. 2).

## 5.2   Numerical sensitivity analysis

A sensitivity analysis of the model (1) is carried out in the sense of [28] and [29] to help us know the parameters that have a high impact on the reproduction number $\mathcal{R}_0$ for effective response to the problem under study.

**Definition 1.** The normalized forward sensitivity index of any given variable $u$ that is differentiable relative to a parameter $b$ is defined as;

$$\Pi_b^u = \frac{\partial u}{\partial b} \cdot \frac{b}{u} \tag{43}$$

The sensitivity indices of the basic reproduction number $\mathcal{R}_0$ with respect to the model parameters are computed as follows:

$$\begin{cases} \Pi_\beta^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial \beta} \cdot \frac{\beta}{\mathcal{R}_0} = 1 \\ \Pi_\tau^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial \tau} \cdot \frac{\tau}{\mathcal{R}_0} = \frac{-\tau}{1-\tau} = -0.25 \\ \Pi_\rho^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial \rho} \cdot \frac{\rho}{\mathcal{R}_0} = 1 \\ \Pi_\gamma^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial \gamma} \cdot \frac{\gamma}{\mathcal{R}_0} = \frac{-\gamma}{\gamma+\mu} = -0.33 \\ \Pi_\delta^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial \delta} \cdot \frac{\delta}{\mathcal{R}_0} = \frac{-\delta}{\delta+\mu} = -0.40 \\ \Pi_\mu^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial \mu} \cdot \frac{\mu}{\mathcal{R}_0} = \frac{-\mu(\delta+\gamma+2\mu)}{(\delta+\mu)(\gamma+\mu)} = -1.26 \end{cases} \tag{44}$$

Where $\mu = 0.016$, $\tau = 0.2$, $\delta = 0.0108$, $\gamma = 0.008$, $\rho = 1.05 \times 10^{-7}$ and $\Lambda = 11,220$. The positive sign of sensitivity index (S.I) of the basic reproduction number $\mathcal{R}_0$ to the model parameter indicates a direct variation relationship, that is an an increase or (a degrease) in the respective parameter causes a corresponding increase or(decrease) on the reproduction number. On the other hand, the negative sign of S.I of the basic reproduction number to the model parameters implies inverse proportionality relationship, that is a decrease or (increase) in a given model parameter causes a corresponding increase or (decrease) on the reproduction number. Hence, with sensitivity analysis, it's possible to get insight into the appropriate intervention strategies to prevent and control the spread of mathematics anxiety influence on mathematical potential students.

## 5.3   Numerical simulations

The numerical simulations of system (1) is presented to illustrate our results using Matlab inbuilt ode solver. Using the parameters value in Table 2, with $\tau = 0.6$ and different initial values for each variable P(t), $A_x(t)$ and $A_a(t)$, we have the anxious-free equilibrium $E^0 = (418657, 0, 282593)$ and so $\mathcal{R}_0 = 0.732649253 < 1$. It is evident from figure 3, that the solution profiles of system (1) converges to the math anxious-free equilibrium in all the two different initial values of P, $A_x(0)$ and $A_a$. This confirms the analytical results of the local as well as the global asymptotic stability of the math anxious-free equilibrium. It is also evident from the figure 3 and 4, that with high value of $\tau$ ( the level of how well an individual understands mathematics algorithms), the total number of math anxious individuals drop to 0 with time.

Similarly, using the parameters value in Table 3, with $\tau = 0.2$ and different initial values for each variable P(t), $A_x(t)$ and $A_a(t)$, we have the endemic equilibrium $E^* = (285714, 148453, 267083)$ and $\mathcal{R}_0 = 1.465298507 > 1$. it is seen clearly from figure 2, that the solution profiles of system (1) converges to the endemic equilibrium $E^*$ in all the two different initial values of P, $A_x$ and $A_a$. This confirms the analytical results of the local as well as the global asymptotic stability of the endemic equilibrium. Furthermore, using the parameters value in Table 2, with $\tau = 0.4540361597$, so that $\mathcal{R}_0 = 1$, it is clearly seen from figure 4, that the solution profiles of system (1) gradually at long run converges to the math anxious-free equilibrium in all the two different initial values of P, $A_x$ and $A_a$. From figure 4, it is revealed that $\mathcal{R}_0 = 1$ is the margin between math anxious-free and endemic equilibrium. Thus, the analytical results of the local as well as the global asymptotic stability of the math anxious-free equilibrium are therefore, confirmed.
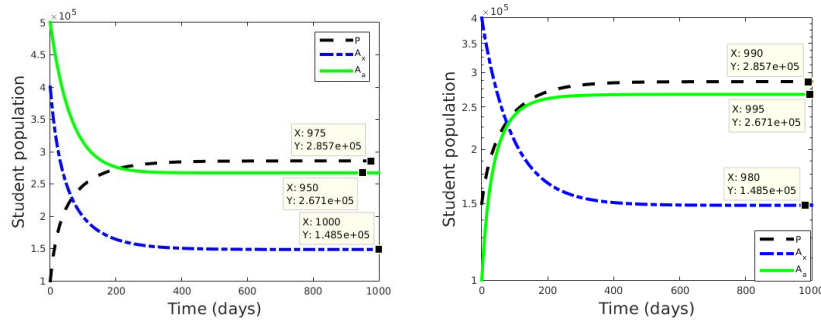
**Fig. 2. Time history and phase portraits of system (1) for** $\Lambda = 11,220$, $\delta = 0.0108$,
$\mu = 0.016$, $\gamma = 0.008$, $t_f = 1000$ **days,** $\tau = 0.2$, $\rho = 1.05 \times 10^{-7}$ **when** $\mathcal{R}_0 = 1.465298507 > 1$
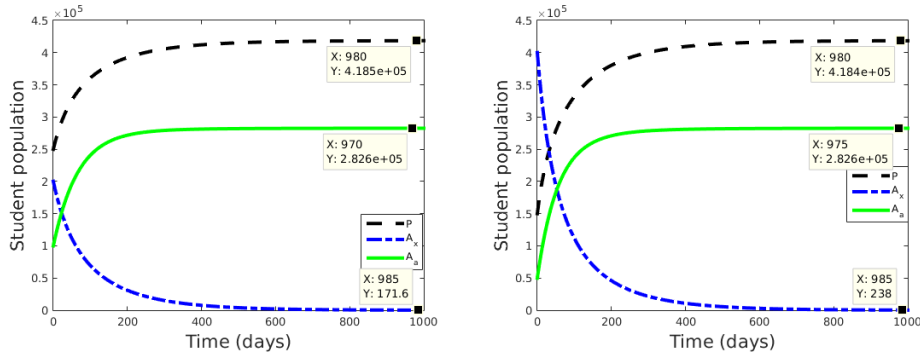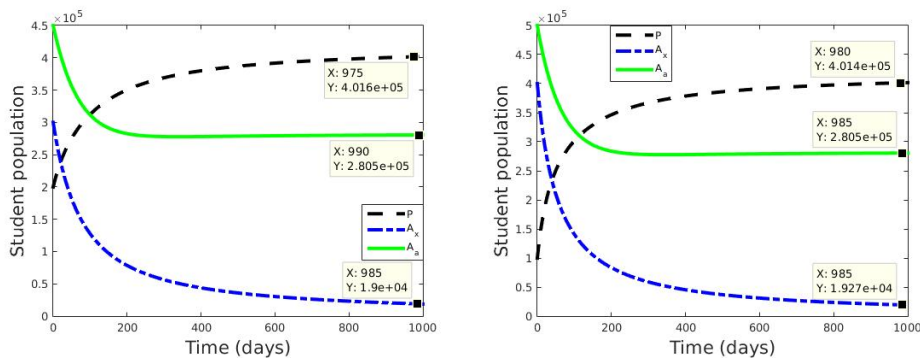**for different initial values for each variable P(t),** $A_x(t)$ **and** $A_a(t)$



**Fig. 3. Time history and phase portraits of system (1) for** $\Lambda = 11,220$, $\delta = 0.0108$,
$\mu = 0.016$, $\gamma = 0.008$, $t_f = 1000$ **days,** $\tau = 0.6$, $\rho = 1.05 \times 10^{-7}$ **when** $\mathcal{R}_0 = 0.732649253 < 1$,
**for different initial values for each variable P(t),** $A_x(t)$ **and** $A_a(t)$



**Fig. 4. Time history and phase portraits of system (1) for** $\Lambda = 11,220$, $\delta = 0.0108$,
$\mu = 0.016$, $\gamma = 0.008$, $t_f = 1000$ **days,** $\tau = 0.4540361597$, $\rho = 1.05 \times 10^{-7}$ **when** $\mathcal{R}_0 = 1$. **for**
**different initial values for each variable P(t),** $A_x(t)$ **and** $A_a(t)$

# 6   Conclusion

The compartmental model with constant recruitment rate for the transmission dynamics of math anxiety as an infectious disease was proposed in this work. The basic reproduction number $\mathcal{R}_0$ was obtained and the analysis revealed that for $\mathcal{R}_0 \leq 1$, the math anxious-free equilibrium is globally asymptotically stable. If $\mathcal{R}_0 > 1$ the math anxious-free equilibrium point is unstable and the endemic equilibrium emerges. Furthermore, sensitivity analysis shows that achievement rate of potential achievers $\delta$ and achievement rate of math anxious students $\gamma$ are the most sensitive parameters. This indicates that effort should be directed towards these parameters, by having qualified and well trained mathematics staff and the best printed and technological resources so as to control the spread of mathematics anxiety. Furthermore, scaling up the understanding level of mathematics algorithms(i.e allowing students to create their own methods to resolve issues stretches their thinking and analytical skills), lowers the mathematics anxiety level and consequently, the spread of mathematics anxiety amongst students reduces. Developing optimal control strategies is left for future work.

# 7   Recommendation

Although the ministry of education has adopted various strategies in countering mass failure in mathematics, the use of well trained mathematics teachers to accelerate recovery of mathematics anxious students holds the great promise in reduction of burden of mass failure in mathematics in Kenya. Two of the key indicators of deep learning and conceptual understanding are the ability to transfer knowledge learned in one task to another task and the ability to move between different representations of mathematical objects. Failure of any given system to achieve these two indicators, exposes the potential learners to anxiety influences. To increase the probability of success against math anxiety influence then the ministry of education and other stakeholders need to address the following factors; First,the qualification and training process of mathematics teachers is paramount in overcoming this problem. Higher level of qualification on tutors side make it easier for them to relate a particular learnt skill to its relevant area of application (more studies should be done to know the minimum training threshold for a mathematics teacher). Secondly, the printed media need to be reviewed and presented in a way that it links the theory learnt in class to real life situations or area of application. Lastly, technology platforms offer a number of didactic advantages that can be exploited to promote a more active approach to learning of mathematics, where students become involved in the discovery and understanding process, rather than viewing mathematics as simply receiving and remembering of algorithms and formulae.

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## Competing Interests

Authors have declared that no competing interests exist.

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