MATHEMATICAL MODELING OF BURGLARY DYNAMICS INCORPORATING UNEMPLOYMENT IN KENYA

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TITLE PAGE

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DEDICATION

To my loving and supportive parents Mr. Peter Okuna Singa and Mrs Siprose Okuna , my siblings Basil, Brian, Taphy and Darline, I dedicate this work for their support and encouragement throughout the course. Without their support it may not have been easier to accomplish this. Most importantly I dedicate this project to the Almighty God for always guiding and strengthening me.

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ABSTRACT

Burglary remains a significant concern in Kenya, affecting the country's economy and social fabric. It is often driven by the perceived presence of valuable commodities in targeted premises and has been linked to other crimes such as rape, arson, and others. The key factors contributing to burglary in Kenya include poverty, unemployment, corruption in the criminal justice system, peer pressure, drug abuse, and high levels of education, among others. According to the December 2023 report by the National Crime Research Center, unemployment was identified as the leading contributor to burglary incidences. The financial constraints resulting from unemployment increase the likelihood of individuals resorting to burglary as a means of meeting basic needs and financial obligations. Unemployment increases the chances of people turning to burglary due to the lack of a legitimate income source. This study formulated and analyzed a deterministic mathematical model that described the dynamics of burglary as influenced by unemployment in Kenya using ordinary differential equations. The model solution was shown to be positive and bounded, confirming its well-posedness. The existence of steady states was analyzed and the effective reproduction number was derived using the next-generation matrix approach. The burglary-free equilibrium was proven to be locally and globally asymptotically stable when $R_e < 1$, while the endemic equilibrium was locally and globally asymptotically stable when $R_e > 1$. The sensitivity analysis of R_e with respect to the model parameters was carried out using the normalized forward sensitivity index, which showed that the contact rate between susceptible individuals and burglars τ and the rate at which susceptible individuals became burglars upon contact δ were the least sensitive parameters, while the employment rate ω was the most sensitive parameter. The lower the rate of employment in Kenya, the higher the prevalence rate of burglary in the human population. Numerical simulations of the developed model were performed using ODE 45 solver in MATLAB software, and the results demonstrated that a high employment rate or the creation of job opportunities among youth drastically reduces burglary incidences. The findings of this study offer valuable information to decision makers in the National Police Service and other security agencies, highlighting critical areas for intervention to curb burglary in Kenya. Furthermore, this work contributes to the broader field of social mathematical modeling by providing a framework for analyzing crime dynamics resulting from socioeconomic factors.

TABLE OF CONTENTS

List of Tables

List of Figures

List of Abbreviations

ODE	:	Ordinary Differential Equations
$\mathbf{R}_e$	:	Effective Reproduction Number
BFE	:	Burglary Free Equilibrium
BEE	:	Burglary Endemic Equilibrium
MATLAB	:	Mathematics Laboratory

CHAPTER ONE

INTRODUCTION

1.1 Background Information

Crime is an unlawful act that constitutes an offense and is therefore punishable by law. Crime is also defined as intentionally committing an act which is socially harmful and is precisely defined, forbidden under the law, and also punishable according to Mataru [10] and musoi [18] . According to Siegel [24], crime is a violation of the societal rule of behavior expressed and interpreted by a criminal legal code created by people holding economic, social and political power.

Crimes are classified into four main categories namely; felony, misdemeanor, petty offenses, and capital offenses [1]. Crimes such as arson, manslaughter, kidnapping, abduction, cybercrime, drug possession, sex crimes, rape, assault, affray, riot, forgery, creating disturbances, offensive conduct, malicious damage, burglary are classified as felony [1]. Felony are crimes that are punishable without proof of previous conviction with death or imprisonment. Petty theft, simple assault, trespass, and careless driving are categorized as misdemeanor crimes. The misdemeanor is a lesser crime punishable with a fine or three years imprisonment. Petty offenses are offenses that are punishable by a small fine, warning, community service, or less than one year imprisonment and are less severe compared to misdemeanor and felony, examples of petty offense are obtaining property by false pretense, creating public nuisance, loitering with intent to commit prostitution, public intoxication, being drunk and disorganized, hawking without permit, loitering, littering, among others. Capital offenses are more serious offenses that are punishable by death; examples are murder, homicide, treason, robbery with violence, espionage [1]. Burglary is an act in which an offender unlawfully breaks into a building, a home, or other premises with the intention of stealing. According to the Kenyan Penal Code 2009 chapter 63, section 304; burglary, housebreaking and similar offenses subsection (2), burglary is breaking into a building, tent, or any other premise with the intention of committed a felony or having committed a felony in such a premises breaks out there at night. The seriousness of the offense with regard to burglary is reflected in the punishment such as seven-year imprisonment [1].

The desire of a country to provide maximum protection to its citizens is reflected in the extent of punishment that is imposed on a person who violates the security and privacy of a given premises. The seriousness of the offense with respect to burglary is reflected in the punishments. Burglary is a common property crime that occurs everywhere in the world. In recent decades, it has been a major problem in Kenya, for example, burglary cases that are being recorded on a daily basis are on an increasing trend in both rural and urban areas [14]. This reflects that the number of burglary incidents is high nationally. It imposes enormous financial and social problems on victims. At some level of argument, the insurance companies does not provide comprehensive cover to apartments or home owners, this increase in premium for comprehensive cover has accelerated burglary incidences in the country. For the burgled individuals, there is always an important cost they undergo apart from financial lose; emotional and psychological stress. When an individual finds that his or her house has been ransacked and the valuable properties either strewed in the house or missing, it creates outrageous feelings, frustration and exposure which has a long lasting effect on health and mental issues. These can leads to psychological and emotional damage [13].

Burglary in urban areas in Kenya spreads like an infectious disease than in rural areas. Burglars take advantage of the nature of settlement in urban settings, and once they have committed a crime, they disappear easily due to congestion and many corridors that police officers cannot follow freely to arrest them. Burglars use four constructive ways to gain access to human dwellings or any other premises that include threats, artifice, collusion, and aperture. Using word of command to gain entry into buildings or any premises is known as threat, artifice is where an offender uses a trick, collusion is an agreement between an inmate of the house and the offender, while aperture is an opening of the building which can be used by an offender to gain entrance [1]. Burglary continues to be a problem that affects peace and security in Kenya.

According to the report of the Research and Information Center [11] on crime with the support of the government. In the recent decade, there has been an increase in professional burglars making a living from practicing burglary [14]. These professional criminals have been on the rise due to economic stress in Kenya. In Kenya, more than one million people graduate from higher learning institutions annually while market demand remains constant or decreases [4]. These graduates continue to add to the number of unemployed youth in the country, yet they have to find ways of survival in an economy. Due to low wages, some of them find themselves committing crimes in order to meet their basic needs and financial obligations. When people are underemployed or completely jobless they are likely to engage in property crime like burglary because at the end, they will make a living even if it is illegitimate since money is the main motivating factor [25]. Due to the interaction between individuals who have made it through committing burglary and unemployed individuals, the unemployed are likely to be influenced to commit property crime such as burglary. From the economic theory of crime developed by [2], it is assumed that most people tend to engage in criminal activities when the benefits gained are greater than the cost of committing a crime, or if the cost of carrying out imprisonment is high, parole elasticity of responses of the offenses by police officers is costly and, therefore, offenders take advantage to commit crime [2].

The incidence of burglary in Kenya increases daily due to the high level of unemployment among youth [5]. For instance, most youths move to urban areas to look for better jobs, unfortunately not all of them are absorbed in various ministries and companies. Thus, most of them find it difficult to survive in an economy, this condition triggers some of them to engage in property crime such as burglary to meet financial obligations [13].

In every economic growth, youth are very important assets in relation to the growth of a nation. Without youth empowerment, there are high chances that the incidence of crime increases in the country due to economic stress. Most youth find it difficult to survive after graduation as the demand for the labor market is lower than the labor supply in Kenya [19]. Therefore, a big number end up jobless or under-employed. Youth who have adequate skills in computing and other fields at some time engage in criminal activities and become professional criminals, and dealing with such professional burglars or criminals becomes too expensive for nations [14]. In rural areas and some parts of urban areas such as slums, unemployment increases rapidly since many children that are raised in such areas do not excel academically due to lack of school fees and other basic needs. Therefore, some of them drop out of school early, while others finish high school but do not join higher institutions of learning. All these people are forced to struggle for survival in a resource constrained, hence some of them end up engaging in criminal activities such as burglary [13].

A sample of data from the National Crime Research Center is shown below in Table 1.1[11].

Table 1.1: **Factors contributing to burglary**

Factors	**Frequency**	**Per cent of cases**
Unemployment	4244	80.0
Availability of illicit drugs	3572	70.1
Idleness	3243	63.4
Poverty	2687	52.9
Corruption in criminal justice	1165	30.5
Ignorance of law	1032	25.3
Physical Environmental factors facilitating crime such as lack of street lights	1001	20
Lack of professionalism	589	14.4
Porous borders	209	4.7
Business rivalry	158	3.6
Psychological disorder	144	2.7
Educational level	89	1.9
Greed	70	1.6

The December (2023), report from the National Crime Research Center, has shown that unemployment goes hand in hand with other factors that are more likely to accelerate criminal activities in Kenya [25]. From the report, it is clear that a lack of financial stability might lead someone to engage in criminal activities as an alternative way to earn a living. However, unemployment can be linked to other causative factors that have a more direct effect on criminal activity. Studies have shown that unemployment is directly related to poverty, exposure to a violent environment, psychological stress, corruption, and others factors that leads criminality. Among 80% of the incidence of burglary that has been reported and recorded, 62. 5% is constituted by unemployment [25].

In this study, we sought to develop a mathematical model to study the dynamics of burglary incorporating unemployment in Kenya.

1.2 Statement of the Problem

The Government of Kenya, through its security agencies in collaboration with non-governmental organizations, has reiterated its commitment to ensure the security of all citizens [4]. Security is a state where the human being and its properties are free from any danger and threats. Factors such as unemployment, poverty, inadequate health services and inadequate infrastructures are some of the threats that lead to insecurity in Kenya [12]. For instance, burglary as a result of unemployment has escalated in the recent past and has led to increase in insecurity in both urban and rural areas in Kenya [4]. Burglary has different dynamics and varies from one geographical area to another. Statistical studies done on the prevalence of burglary due to unemployment in India [23] showed that most youth tend to engage in criminal activities when the benefits gained is greater than the cost of committing a crime. This showed that there is a direct link between burglary and unemployment. With the continuous trajectory of the increase of unemployed individuals in Kenya, burglary cases is projected to increase in the near future; thus the need to develop a mathematical model of dynamics of burglary as a result of unemployment by the use of ordinary differential equations to identify the most sensitive parameters to curb the trajectory since none of the studies that have been previously developed explicitly describes it mathematically.

1.3 Objectives of Study

1.3.1 Main Objective

To develop and analyze a mathematical model of burglary dynamics incorporating unemployment in Kenya.

1.3.2 Specific Objectives

The study was guided by the following specific objectives;

(i) To formulate a mathematical model of burglary dynamics incorporating unemployment in Kenya by use of a system of ordinary differential equations (ODE).

(ii) To carry out stability analysis of the model in order to study the behavior of the solutions around the equilibrium points.

(iii) To perform numerical simulation by use of MATLAB software in order to establish the long term behavior of solutions of the model developed in (i)

1.4 Justification of the Study

Unemployment remains a critical socio-economic challenge affecting many developing countries, included Kenya. It serves as a key indicator of economic health of a nation, with widespread unemployment often associated with adverse social consequences [17]. In Kenya, the persistently high unemployment rates have exacerbated economic instability and contributed to a rise in criminal activities, particularly burglary. Burglary has become a pervasive issue, threatening both rural and urban areas, and poses a significant challenge to national security[19].

This study is justified by the need to establish a scientific understanding of the interplay between unemployment and burglary. By employing a system of ordinary differential equations, the study provides a mathematical framework to analyze the dynamics of burglary as a result of unemployment. This approach offers valuable insight into the mechanism driving crime rate and provides empirical evidence to guide policy interventions. Addressing these interlinked issues is essential for fostering economic growth, enhancing security and improving the overall quality life in Kenya[14].

1.5 Significance of the Study

The study has significant for several stakeholders and domains, for instance policy and security agencies; the finding of the study provide decision makers such as National Police service with insight to identify priority areas for crime prevention and resources allocation. The establishment relationship between unemployment and burglary helps inform targeted strategies and burglary incident in both urban and rural areas. The study offer valuable insight for organization addressing socio-economic challenges. By identifying unemployment as a potential driver of burglary. These organizations can design effective program and policies to mitigate both unemployment and its associated social consequences. Academic contribution; the study contributes to the academic field of crime and security studies by offering a novel mathematical perspectives on the relationship between unemployment and burglary. Scholars and researchers can build upon this work to explore further socio-economic factors influencing crime dynamic. Social mathematical modeling; this study enhances the growing body of knowledge in social modeling by demonstrating the application of ordinary differential equations to real world socio-economic problems. It provide a framework for analyzing crime dynamics thereby advancing the interdisciplinary field of mathematics and social science.

1.6 Methods

The following methods are used to achieve the objectives of the study.

1. Burglary dynamic model in the form of non-linear system of ordinary differential equation (ODE) is developed and analyzed.

2. Stability of the equilibrium points of the developed model is carried out. This involves linearizing the model about equilibria points and analyzing stability of the equlibrium points of the linearized system. The Routh-Hurwitz criterion is applied to analyze the local stability of Burglary Free Equilibrium BFE and the Endemic Equilibrium EE. The linear Lyapunov technique is applied to analyze global stability of BFE and logarithmic Lyapunov technique is used to analyze global stability of EE. Sensitivity analysis is carried out by the use of by use of normalized forward sensitivity index and effective reproduction number R_e is computed using the next generation matrix

3. Numerical simulation is carried out using ODE 45 solver in MATLAB software to graphically illustrate the behavior of the solutions of the model.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter provides a theoretical and empirical evaluation of previous studies related to the link between unemployment and burglary. It explores mathematical models that have been applied in analyzing crime dynamics and unemployment trends across various economies, with a particular focus on third-world countries. By reviewing relevant literature, it aims to establish the relationship between unemployment and crime, highlighting key factors that contribute to this social challenge.

2.2 Unemployment and Burglary

Unemployment refers to a condition in which an individual, aged eighteen years and above, is both willing and able to work but remains without a job despite actively seeking employment. It is characterized by a discrepancy between wages and rewards, which often fall below the required standards for sustainable living. High unemployment rates arise when the growth of the workforce surpasses the availability of job opportunities [5], leading to increased economic instability and heightened crime rates.

In every economic growth, youths are very important asset in relation to the growth of a nation [5]. The future of a nation is always directly proportional to it's government or youths, therefore, for economic stability, it is very important for a nation to empower youths. This greatly increase the likelihood of a nation since youths are viewed as the engine for economic growth. Investing on youths enable them to

acquire appropriate skills that can meet market demand and as well as start and run their own business, hence resulting to a greater reduction of any threats or insecurities of a nation. In 21st century (today) youths all over the world face a big challenge known as unemployment, which has been linked with inappropriate skills and low education system to meet the market demand, hence low employability. In this scenario, youths are the most affected people by the economic decline.
Unemployment has posed significant challenges across many economies worldwide. Governments frequently rely on unemployment rates to assess a nation's overall economic well-being and public satisfaction, including in Kenya. There are four primary types of unemployment identified in Kenya: frictional, structural, cyclical, and seasonal.

Frictional unemployment arises due to temporary shifts in individuals' careers, such as relocating to a new area or recent graduates entering the job market. Often, individuals who leave one job in search of another may experience difficulty securing employment, leaving them jobless for a period. This, in turn, can lead some individuals to engage in unlawful activities to meet financial needs. Structural unemployment occurs when there is a mismatch between the available workforce and the requirements of existing job opportunities. In some cases, job seekers lack the necessary skills, while in others, skilled workers face a shortage of available positions. Cyclical unemployment results from reduced labor demand, often caused by insufficient consumer demand for goods and services. When businesses struggle to generate revenue, they cut back on employment opportunities, leading to job losses and an increased crime rate. Seasonal unemployment, on the other hand, arises due to variations in labor demand across different times of the year. This type of unemployment significantly affects Kenya's economy, influencing wage trends and financial stability. According to [5], unemployment in Kenya remains one of the key economic factors impacting wage level and overall financial conditions.

Kenya's unemployment has been a challenge since independence (1963). Currently, frictional and seasonal are the core types of unemployment that is affecting the country[5]. The causes and the nature of this problem are corruption, economic decline, seasonality of industries, rapid growth of labor force, job selectiveness by graduates, improved technology and others. These conditions have impacted the country's growth in multiple ways, particularly through reduced economic efficiency. During periods of high unemployment, many job seekers are forced to accept positions below their qualifications, a phenomenon known as underemployment. The resulting decrease in consumption among unemployed individuals weakens one of the key drivers of economic growth, leading to further economic decline. This cyclical pattern limits opportunities and may push some individuals toward criminal activities such as burglary. The effects of unemployment manifest at both individual and socio-political levels, creating a complex challenge for national development.

The consequences of unemployment extend across multiple dimensions of Kenyan society. At the individual level, joblessness severely compromises people's ability to meet financial obligations, often leading to deteriorating mental health, increased illness rates, and in extreme cases, homelessness. This downward spiral in living standards contributes significantly to the nation's rising poverty levels. Desperate for survival, many unemployed youths resort to criminal activities, particularly burglary, to generate income. On a broader scale, sustained high unemployment breeds social and political instability. When citizen dissatisfaction reaches critical levels, it frequently erupts into civil unrest. During such periods of turmoil, opportunistic individuals often exploit the chaos to loot homes and businesses, further compounding the country's security challenges [19].

The economy of Kenya, currently was hit by several shock one after the other[19]. Firstly, skyrocketing of fuel price due to increased landing cost of imported fuel from July 2023 to date,has directly affected the economy, since retail consumer price has gone high. This has directly affected business and companies resulting in joblessness in the country since many companies collapses and the few remaining cannot absorb many graduates, Secondly, the boom and cycle exportation of coffee and tea to UK. The decline in the exportation of the above products have greatly affected the Kenyan economy since 2020. Many people who were employed in tea and coffee farms lost their jobs and even small scale farmer who use to receive bonus every end year could not get it anymore, this resulted to a decrease in consumer's expenditure hence economic decline. Some People who lost their jobs are likely to engage in criminal activities such as burglary to attain their basic needs and financial satisfaction. Thirdly, the outbreak of COVID-19 pandemic, the coronavirus affected the operation of the retailers, businesses and companies at large in the world, Kenya included. Many people remained jobless since companies followed the restrictions that were given by World Health Organizations. Up to now many employees have not resumed or found other job opportunities. Some of these jobless people are forced to commit crime such as burglary to meet their financial goals [19].

Unemployment coupled with shortages of basic necessities like food significantly drives rising crime rates in Kenya [19]. Empirical studies, including research by [5], demonstrate a strong link between joblessness and criminal behavior. Crossnational analyses of socioeconomic conditions and crime patterns reveal that Kenya, like other developing nations, experiences elevated crime rates when economic indicators worsen particularly when incomes decline, employment opportunities shrink, and governance systems falter.

The Kenyan government since independence has made various policies to solve this issue of unemployment. The current Kenya Kwanza administration's affordable housing initiative, for example, aims to create youth employment through construction projects. However, its urban-centric focus limits job creation, absorbing only a fraction of the workforce while leaving most young people unemployed [9]. These jobless youths are always at risk and most of them are likely to be recruited to burglary [9]. Education and technical training have been given priorities to input appropriate skill on youths so that they can meet the market demand nationally and internationally. However, Kenya lacks adequate companies and industries for attachments where trainees can gather more skills and knowledge necessary for professional world market. This has led to lack of required practical skills and experience that is needed for a given task. Therefore, unemployment situation increases gradually due to lack of practical skills. The coordination between education, technical institution and industries is vital for the skill development that later leads to employability among youths in Kenya [9]. Furthermore, inflationary pressures as noted by [9] reduce consumer spending power, depressing business growth and further limiting job creation.

A core objective of governance and political systems is to enhance the influence and standing of individuals or societal factions [21]. Those in privileged positions often aggressively pursue dominance within power hierarchies, frequently exploiting marginalized groups to advance their agendas. This systemic manipulation typically involves shaping public perceptions of governance to align with the interests of entrenched political elites, perpetuating cycles of inequality.

Consensus approach involves smooth process of formulating and implementation of laws to accommodate the society's interests in the development of agreeable system of governance regardless of the society's dynamics. According to the second model of deviants, some actions seem to pose greater threats to the survival of society and are considered forms of crime[21]. A wrong or right action is determined by the society. This creates social control mechanisms for settling disputes arising from deviant behaviors. The above statement reinforces the assumption that members of the ruling class are the major custodians of powers to convert their interests into criminal laws for protecting their wealth. They are powerful enough to alter ways in which criminal laws are enforced and administered in the society.

The foregoing notion can only be changed by changing the superstructure to include everyone's interests. It is observed that members of the ruling class influence every stage of defining crime and criminal laws. Therefore, crime turns to be a political behavior while criminal law becomes a burden to less privileged groups with no ability to influence law enforcement. However, [6] observed that judicial activity and efforts to enforce criminal law can be increased by raising an opposition strong enough to threaten the ruling class interests [6] . Academic research offers multiple crime prevention strategies, including neighborhood watch programs where organized community members provide supplemental security patrols, particularly in areas with limited police presence [7,11]. These initiatives have proven effective in deterring burglary and other property crimes through localized surveillance [10]. A parallel approach, community policing, emphasizes collaborative partnerships between law enforcement and residents. This strategy operates on the premise that community members possess vital situational knowledge about criminal activity in their neighborhoods, enabling more targeted interventions when combined with police resources.

A complementary approach to neighborhood watch initiatives involves community policing strategies. This model establishes formal partnerships between law enforcement agencies and local residents to jointly address criminal activity [6]. The methodology relies on two foundational principles: (1) community members typically possess superior knowledge about local criminal networks, and (2) timely information sharing with authorities enables more effective offender apprehension and prosecution. Research demonstrates this collaborative approach significantly reduces neighborhood crime rates when properly implemented [6]. Mburu [13] argued that offenses against property such as burglary are by far the most numerous of all crimes that violate the law in nearly all countries in Africa. Cases of armed robberies, theft, pick pocketing and stealing, theft of foodstuffs, malicious damage to property are as a result of burglary at some point and are too numerous to enumerate. As the nation advances, there is an upsurge of cases such as burglary in both rural and urban areas, to secure cash or physical objects. The increase of cases of burglary and other property crimes has led to the introduction of the death penalty in a number of African countries among them Uganda, Kenya, Zambia, and Nigeria

The consistent presence of law enforcement personnel through scheduled patrols serves as an effective deterrent against burglary and related criminal activities.

Likewise, presence of plainclothes police squad is an important deterrent of crime because the officers are able to collect vital criminal intelligence without being recognized and thus detect offenses. However, there are challenges that are encountered by police officers while enforcing the law in the urban areas. These include inaccessibility of living quarters and lack of security ans street lights during the night. Most areas are filthy and dangerous making it difficult for extensive police foot or mobile patrols to be carried out. Crime prevention may be achieved in Kenya through ability of police to identify crime hot-spot areas where citizens are often attacked by criminals. The police should also be the citizens guardians to protect them through intelligence, led patrols and engage cooperation of residents through community policing. The diversity of approaches includes engagement of faith-based organizations, non-governmental organizations and county government agencies in addressing the factors that contribute to crime in the community for instance improving infrastructure, putting up street lights, rehabilitating delinquent youths and reducing social disorder which include sensitizing the youth against abuse of drugs and alcohol.

2.3 Crime Mathematical Models

Mathematical modeling has emerged as a crucial analytical tool for examining complex social issues, including burglary patterns. These quantitative approaches provide valuable insights for policymakers developing intervention strategies to combat criminal activity. Compartmental modeling techniques have proven particularly effective in bridging mathematical analysis with sociological research, offering innovative solutions to societal challenges. The application of mathematical models to crime analysis represents a growing interdisciplinary research field. Short et al. [23] pioneered this approach by creating the first statistical framework for burglary, integrating criminological theory with quantitative methods. Subsequent studies have expanded this foundation, developing both linear and nonlinear models that examine how criminal behavior spreads through social networks and peer interactions.

A study by [31] added that poverty has a direct correlation with criminal activity, building on Becker's foundational theory, which asserts that individuals facing economic deprivation are statistically more likely to engage in property crimes due to the potential material gains. In addition, they suggested that the rehabilitation and incarceration program can be used to reduce the crime rate. However, the program is too expensive for every country compared to eradication of poverty. But it is obviously known that crime can either be alleviated by increasing the cost of punishments or reducing the poverty level. Even though the world resources are ever limited, therefore poverty cannot be relieved fully and all criminals cannot be incarcerated[31]. The model optimized intervention that control the cost of reducing crime level. The γ and ρ were the interventions where γ is the rate at which individuals transits from poverty class to recovered class and ρ the incarceration rate of offenders. These variables dynamically adjust based on enforcement resource allocation. The model framework divides the total population into five distinct socioeconomic categories: The impoverished class N, poverty class P, criminal class C, jailed class J, recovered class from jail or from impoverished class$R$.
Population transitions between these states are governed by several rate parameters. The transition from non-impoverished to impoverished status is denoted by σ. The rate at which an individual move from poverty class to recovered class due to government intervention was denoted by γ, the rate at which criminals are captured was ρ. The proportion of criminals entering the justice system is denoted by ε, prison release rate is denoted by δ and baseline population turnover accounting

for death rates was given by μ while birth rate is denoted by Λ. The complete system dynamics are mathematically represented through a series of interconnected differential equations is as shown below.

$$\begin{aligned}
\frac{dN}{dT} &= \mu T - (\sigma + \mu)N \\
\frac{dP}{dT} &= \sigma N - \beta PC - (\gamma + \mu)P \\
\frac{dC}{dT} &= \beta PC + \phi\beta RC - (\varepsilon\rho + \mu)C \\
\frac{dJ}{dT} &= \varepsilon\rho C - (\delta + \mu)J \\
\frac{dR}{dT} &= \gamma P + \delta J - \phi\beta RC - \mu R
\end{aligned} \tag{2.1}$$

According to [31] all crimes are grounded or coursed by poverty, however in most cases not all crimes are constituted by poverty. The effect of unemployment on criminal activities is almost the same as that of poverty. The model developed by [31] helped us to study the behavior of burglary in Kenya as a results of unemployment.

Research conducted in West Malaysia by [22] established a significant correlation between socioeconomic deprivation and criminal. he Malaysian government has made poverty alleviation a central focus of national policy development. Their approach involves establishing an official poverty line that measures minimum consumption requirements for basic necessities including adequate shelter, sufficient food, and proper clothing. This economic threshold, determined through comprehensive household income analysis, serves to identify two distinct disadvantaged populations: individuals marginally below this line classified as living in poverty and those significantly below it, classified as living in extreme poverty. The study's findings indicate that both population segments show substantially higher probabilities of participating in criminal activities, with particular prevalence in both violent crimes and property-related offenses.

The primary objective of this research was to develop a comprehensive poverty-poor-crime model that would effectively demonstrate the impact of government intervention strategies on both poverty reduction and crime prevention in West Malaysia. The analytical framework divides the total population T into three distinct categories: the poverty class L, the poor class H, and the criminal class C. The model incorporates several critical demographic parameters: the birth rate μ_1 , death rate μ_2, the transition rate from poverty to poor status ω, the criminal participation rate among the poor class α, and the criminal participation rate among the poverty class β [22]. These parameters collectively form a system of equations that mathematically represents the complex relationships between socioeconomic conditions and criminal behavior patterns, providing policymakers with a valuable tool for evaluating and optimizing intervention approaches, activity through a dynamic poverty-crime model as shown below.

$$\begin{aligned} \frac{dL}{dt} &= \mu_1 T - \mu_2 L - \omega L - \tfrac{\beta LC}{T} \\ \frac{dH}{dt} &= -\mu_2 H + \omega L - \tfrac{\alpha HC}{T} \\ \frac{dC}{dt} &= \tfrac{\alpha HC}{T} + \tfrac{\beta LC}{T} - \mu_2 C \end{aligned} \tag{2.2}$$

The model demonstrated that decreasing the rates at which impoverished and poor individuals engage in criminal activities represented by parameters α and β, leads to a corresponding reduction in overall crime rates within Malaysia. This phenomenon occurs exclusively when robust government intervention programs are implemented effectively. The analysis further revealed that poverty levels and unemployment rates exert nearly identical influences on criminal behavior patterns within society.

Complementary research by [17] examining optimal control strategies within an unemployment mathematical model established that well-designed government policies focused on youth empowerment and job creation significantly decrease unemployment rates, as evidenced by their case study in India [17]. They suggested that creating more vacancies can be effective in reducing unemployment problem in every country. Factors such as inappropriate skills, physical and mental disabilities, age, high inflation, depression, increase in gross domestic product and others has contributed to this problem in India. Closing down of sick industries left very many employees jobless. Even though technical advancement has promoted the economic growth in the world, uncontrolled and unplanned technological growth has leads to unemployment[17].

The model done by [17], discussed the dynamics of unemployment only in India, in this model, we sought to examine the impact of unemployment on the dynamic of burglary alone as a crime in Kenya. Unemployment has emerged as a critical issue generating significant apprehension among economic analysts and investment stakeholders due to its substantial financial pressures and systemic burdens. This phenomenon contributes directly to widespread impoverishment, destabilization of political and socioeconomic systems, and reduction in average individual earnings. The analytical framework developed by researchers examines the fundamental drivers of unemployment through three primary workforce categories: unemployed individuals $U(t)$, currently employed workers $E(t)$, and newly available positions $V(t)$ at any given time t. The model incorporates several key dynamic parameters: the constant inflow rate into unemployment Λ, the employment acquisition rate is denoted by κ, mortality rates for both unemployed α_1 and employed/retired death rate is α_2, γ is job termination frequency, new job creation rate denoted by ϕ, and business failure rate due to resource deficiencies denoted by δ [17]. The complete mathematical representation of these relationships is presented in the subsequent system of equations as shown below.

$$
\begin{aligned}
\frac{dU(t)}{dt} &= \Lambda - \kappa U(t)V(t) - \alpha U(t) - \gamma E(t) \\
\frac{dE(t)}{dt} &= \kappa U(t) - \alpha_2 E(t) - \gamma E(t) \\
\frac{dV(t)}{dt} &= \alpha_2 E(t) - \gamma E(t) - \delta V(t) - \phi U(t)
\end{aligned}
\tag{2.3}
$$

Research by [26], which analyzed the impact of rising unemployment on crime rates in Indonesia, expanded upon the framework established in [17]. Their findings indicated that the growth rate of unemployment has a significant influence on the increase in criminal activities, demonstrating a direct correlation between the two variables. The study proposed that addressing unemployment through policies aimed at generating more job opportunities for young people could mitigate this issue. Unemployment has been identified as a major challenge to Indonesia's economy, driven by a population growth rate that exceeds labor market demand. When individuals are unable to secure employment, whether in formal or informal sectors, it can lead to psychological distress, affecting not only the individual but also their family and community. Additionally, prolonged joblessness diminishes professional efficiency, as skills acquired during education may deteriorate over time. This creates a cycle of unemployment, as employers are less likely to hire individuals with outdated skills or insufficient experience.

The frustration caused by unemployment may drive individuals to engage in criminal activities, either directly or indirectly. Without stable employment, people are more likely to interact with criminals due to prolonged idleness and financial strain. Unemployment poses a significant threat to a nation's economic stability, social cohesion, and political development, yet effectively addressing this issue remains a persistent challenge. Research by [17] supports this, demonstrating that rising unemployment rates in Indonesia correlate with an increase in criminal behavior. In this study, we adapted the conceptual framework of unemployment's societal impact to analyze burglary trends in Kenya. Previous research by [17] emphasized that failure to address unemployment leads to broader social issues, including poverty and heightened criminal activity. Building on this work, [26] enhanced the original model by introducing a new compartment C representing the criminal population. Key parameters included; ρ for criminal recruitment, φC for criminals reintegrating into employment, $\alpha_3 C$ for mortality due to criminal activity, and βUC for criminal growth through social interactions [26]. The resulting model is represented by the following equation:

$$\begin{aligned}\frac{dU}{dt} &= \Lambda - \mu_1 \kappa UV - \alpha_1 U + \gamma E - \beta UC + \varphi C \\ \frac{dE}{dt} &= \mu_2 \kappa UV - (\alpha_2 + \gamma)E \\ \frac{dV}{dt} &= (\alpha_2 + \gamma)E - \delta V + \mu_2 \phi U \\ \frac{dC}{dt} &= \rho + \beta UC - \alpha_3 - \varphi C\end{aligned} \tag{2.4}$$

Its observed that criminal behavior spread very fast due to interaction with people who are involved in criminal activities and when the benefits are greater than the punishments. Lack of police force, media coverage and moral activities greatly increases the number of criminal individuals in the society that do not discourage criminal activities.

According to [10], unemployment among youth has contributed to a greater percentage to the causes of crimes in developing countries. More than 35% of young population engaged in criminal activities are as a result of unemployment, these people have lost hope in life and later find themselves in armed robbery, prostitution, treason,espionage and other criminal activities for them to meet their basic need and financial satisfaction. Developing countries like Kenya faces economic, political and social hardship due to corruption, political instability, unequal distribution of resources and these has led to lack of jobs among youths and therefore increases high risk of youths being recruited to criminality compared to developed countries. Unemployment results more in material crime than any other crimes. Their model impact understanding the dynamics and control of criminal activities among youths that are caused by unemployment as a constant parameter.

The research model developed by [10] employs a compartmental approach to analyze unemployment-driven criminality among youth populations. The framework divides the total population $N(t)$ into five distinct groups: unemployed individuals $U(t)$, those susceptible to criminal influences $S(t)$, active criminals $C(t)$, vocational training participants $V(t)$, and employed persons $E(t)$. The model incorporates several key parameters: a constant recruitment rate τ for both educated and uneducated individuals into unemployment, a uniform natural mortality rate μ across all groups, and transition rates between compartments. Specifically, individuals progress from unemployment to criminal exposure at rate ϕ, while engagement in criminal activities occurs at rate β for the unemployed and β(1-θ) for the exposed population. Additional parameters include π for criminal recruitment from unemployment, φ for criminality-related mortality, ω for general vocational training enrollment, σ for unemployed individuals entering training programs, ρ for unemployed individuals transitioning to self-employment after skill acquisition, and ε for criminals reforming through rehabilitation programs. This comprehensive framework is mathematically represented through the following system of equations:

$$
\begin{aligned}
\frac{dU}{dt} &= \tau - \beta UC - (\mu + \alpha + \sigma + \phi)U \\
\frac{dS}{dt} &= \phi U + \beta(1-\theta)SC - (\omega + \mu)S \\
\frac{dC}{dt} &= \beta UC + \beta(1-\theta)SC + (\pi + \mu + \varepsilon + \delta + \varphi)C \\
\frac{dV}{dt} &= \omega S + \sigma U + \delta C + (\rho + \mu)V \\
\frac{dE}{dt} &= \varepsilon C + \alpha U + \rho V + \mu E
\end{aligned}
\tag{2.5}
$$

While research by [10] establishes unemployment as a primary driver of criminal activity, it is important to recognize that numerous offenses occur independently of employment status. These include white-collar crimes such as accounting fraud and tax evasion, violent acts like homicide, property crimes including arson, and various traffic violations. This study specifically focuses on developing a mathematical model to analyze burglary patterns in Kenya, with particular emphasis on unemployment as the key influencing factor.

CHAPTER THREE

MODEL FORMULATION AND ANALYSIS

3.1 Introduction

Most social issues, such as burglary exhibit characteristics similar to the spread of an epidemic. It is widely believed that criminal activities in a community tend to increase when individuals in the population come into contact to those already engaged in crime. In this chapter, the parameters of mathematical model capturing this phenomenon are introduced and analyzed. The model illustrates the transition of individuals between various compartments, representing different states of involvement in criminal activities, and clearly outlines the assumptions underpinning its structure.

3.2 Model description and formulation

This study categorizes the total population $N(t)$ is divided into five compartments namely; Susceptible class $S(t)$, exposed class $E(t)$, burglary class $B(t)$, incarcerated class $I(t)$ and released class $R(t)$ are described as follows; The Susceptible class $S(t)$ are individuals in the society who are not currently burglars but are at risk of being influenced into engaging in burglary. Exposed class $E(t)$ are individuals who have been influenced through social interaction with active burglars but are not yet actively participating in burglary activities. Burglary class $B(t)$ consist of individuals who are actively engaged in burglary and can influence others in the Susceptible or Exposed classes to become burglars through their actions or interactions. their actions or interactions to become burglars. Incarcerated class $I(t)$ consist of individuals who have been apprehended and detained by the law enforcement as a result of involvement in burglary activities. Released class $R(t)$

consist of individuals who have completed their incarceration or jail term from the justice system.

Susceptible individuals are recruited at a constant rate Λ, and upon interaction with burglars, they get exposed to burglary activities at the rate β, whereas exposed individuals join burglary class at the rate α. Burglary individuals once proven guilty of committing such a felony are incarcerated at the rate γ, while some of the criminals die as a result of burglary associated activities at the rate ρ . Individuals convicted due to committing burglary are released at the rate ϕ. Upon release, individuals may either move to exposed compartment at the rate $(1-\theta)$ or re-engage in burglary activities at the rate θ. Natural mortality rate is represented by μ At a given time t the total population is given by ;

$$N(t) = S(t) + E(t) + B(t) + I(t) + R(t)$$

and β which is the force of influence is given by

$$\beta = \frac{\delta\tau(1-\omega)(B(t) + \lambda E(t))}{N(t)}$$

Where δ is the probability at which susceptible individuals become burglars upon contact with burglars, τ is contact rate between susceptible and burglary individuals, ω is the parameter accounting for employment rate which ranges from $(0 < \omega < 1)$ while $(1-\omega)$ is the rate of unemployed and λ is a modification parameter which accounts for the fact that exposed individuals to burglary have a higher probability of influencing others since they freely mingle with people in the population whereas the burglars live a secretive lives and fear being arrested thus minimal interaction with susceptible population. The summary of the model parameters are shown the Table 3.1.

Table 3.1: **Model descriptions**

Parameters	**Model parameters descriptions**
Λ	Natural recruitment rate to susceptible class
β	Rate at which susceptible individuals are exposed to burglary due to interaction
α	The rate at which exposed individuals proceed to burglary class
ρ	Mortality rate of burglars as a result of burglary associated activities
γ	Rate at which burglary individuals are being imprisoned
ϕ	Rate at which incarcerated individuals are removed from prison
θ	The parameter captures recidivism. Is the rate at which criminals resume back to burglary activity after released from prison.
$1-\theta$	Rate at which some released individuals relapses back to exposed class
δ	The probability at which susceptible individuals become a burglar upon contact with burglars
τ	rate of contact between susceptible and burglary individuals
ω	Parameter accounting for employment rate
$(1-\omega)$	Rate of unemployment
λ	Modification parameter accounting for the relative influence of the exposed individuals compared to burglary individuals
μ	Natural mortality rate

3.2.1 Model Assumptions

The following assumptions are used to develop the model;

i. The population under the study is homogeneous

ii. Susceptible individuals cannot commit burglary without exposure.

iii. It is assumed that released individuals cannot return to a naive state of susceptibility. Once someone has been through the criminal justice system, their experience may make them less innocent or unaware of burglary compared to individuals in $S(t)$ class, but can be re-exposed or relapses to burglary activities

iv. There is a constant recruitment into study population.

v. Exposed individuals are not easily recognizable in the population unless they commit a felony;

3.2.2 Flow Diagram

The flow diagram presented in this subsection illustrates the transitions between compartments, as derived from the model assumptions and formulation described previously. The schematic representation captures the movement of individuals across different population groups within the system.

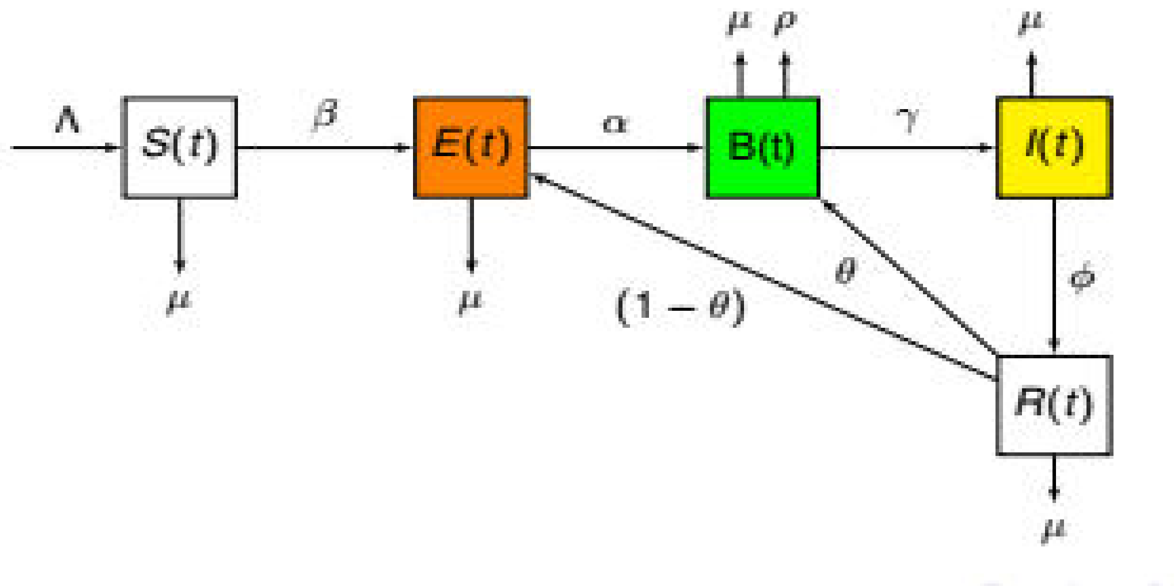

Figure 3.1: Burglary schematic diagram.

From the schematic diagram, the model describing the problem under the study is described in the equation (3.1) below

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - (\beta + \mu)S \\
\frac{dE}{dt} &= \beta S + (1-\theta)R - (\alpha + \mu)E \\
\frac{dB}{dt} &= \alpha E + \theta R - (\rho + \gamma + \mu)B \\
\frac{dI}{dt} &= \gamma B - (\phi + \mu)I \\
\frac{dR}{dt} &= \phi I - (\mu + 1)R
\end{aligned} \tag{3.1}$$

3.3 Model Properties

Analyzing the positivity and boundedness of the model solutions is critical to ensure its mathematical validity and applicability. A model is considered mathematically meaningful if its solutions are both positive and bounded.

3.3.1 Positivity of the Solution

Given that the model under the study describes human population, all the state variables and parameters of the equation (3.1) must remain positive and well defined for all $t > 0$. This ensures that each compartment, such as susceptible, remains physically meaningful throughout the analysis

Theorem 3.3.1. *Positivity*

Let $\Gamma = \{(S, E, B, I, R) \in \mathbb{R}^5_+; S > 0, E > 0, B > 0, I > 0, R > 0\}$. *Then solution of S(t), E(t), B(t), I(t) and R(t) are positive for* $t \geq 0$

Proof. Consider

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - (\beta + \mu)S \\
\frac{dS}{dt} &\geq -(\beta + \mu)
\end{aligned} \tag{3.2}$$

By the use of variation of constant formula, that is integrating both side, the solution of equation 3.2 is as follows

$$
\begin{aligned}
\int \frac{dS}{S} &\geq -\int (\beta+\mu)dt \\
\ln(S)|_0^t &\geq -\int (\beta+\mu)dt \\
\frac{S(t)}{S(0)} &\geq e^{-(\frac{\delta\tau(1-\omega)(B(t)+\lambda E(t)}{N}+\mu)t} \\
S(t) &\geq S(0)e^{-(\frac{\delta\tau(1-\omega)[B(t)+\lambda E(t)}{N}+\mu)t} \geq 0 \text{for} \forall_t \geq 0
\end{aligned} \tag{3.3}
$$

This shows that the state variable $S(t)$ is positive for all $t \geq 0$. Similarly, when the same procedure is applied in Exposed, Burglary, Incarcerated and Released compartments, positive solutions obtained were as follows.

$$
\begin{aligned}
E(t) &\geq E(0)e^{-(\alpha+\mu)t} \geq 0 \text{for} \forall_t \geq 0 \\
B(t) &\geq B(0)e^{-(\rho+\gamma+\mu)t} \geq 0 \text{for} \forall_t \geq 0 \\
I(t) &\geq I(0)e^{-(\phi+\mu)t} \geq 0 \text{for} \forall_t \geq 0 \\
R(t) &\geq R(0)e^{-(\mu+1)t} \geq 0 \text{for} \forall_t \geq 0
\end{aligned} \tag{3.4}
$$

Thus, the proof confirms the non-negativity of the model Equation (3.1), completing the proof. □

3.3.2 Boundedness of the Solution

We show that the model solutions of Equation (3.1) are bounded in the set Γ by adding all the derivative of Equation (3.1) and simplified as follows.

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dE}{dt} + \frac{dB}{dt} + \frac{dI}{dt} + \frac{dR}{dt}$$

therefore

$$\frac{dN}{dt} = \Lambda - \mu S - \mu E - \rho B - \mu B - \mu I - \mu R$$
$$\frac{dN}{dt} = \Lambda - \mu(S + E + B + I + R) - \rho B \tag{3.5}$$

$$S + E + B + I + R = N$$

hence

$$\frac{dN(t)}{dt} = \Lambda - \mu N - \rho B$$
$$\frac{dN(t)}{dt} \leq \Lambda - \mu N \tag{3.6}$$

By the use of variation of constant formula and integrating both sides, the result are obtained as follows.

$$\int_0^t \frac{dN}{\Lambda - \mu N} \leq \int_0^t dt$$
$$\frac{-1}{\mu} ln(\Lambda - \mu N) \leq t$$
$$\frac{\Lambda - \mu N(t)}{\Lambda - \mu N(0)} \leq e^{-\mu t}$$
$$N(t) \leq \frac{\Lambda}{\mu} - \frac{\Lambda - \mu N(0) e^{-\mu t}}{\mu}$$

Let $A = \frac{\Lambda - \mu N(0)}{\mu}$. therefore

$$N(t) \leq \frac{\Lambda}{\mu} - A e^{-\mu t} \tag{3.7}$$

at t = 0

$$N(0) \leq \frac{\Lambda}{\mu} - A e^{0}$$
$$A \geq \frac{\Lambda}{\mu} - N(0)$$

therefore

$$N(t) \leq \frac{\Lambda}{\mu} - (\frac{\Lambda}{\mu} - N_0)e^{-\mu t} \tag{3.8}$$

by applying initial condition as $t \to \infty$ and the total population $N(t) \to \frac{\Lambda}{\mu}$ then

$$0 \leq N(t) \leq \frac{\Lambda}{\mu}$$

$$N(t) \leq max(\frac{\Lambda}{\mu}, N(0))$$

$$N(t) \leq \frac{\Lambda}{\mu}, \text{otherwise} N(0) \text{is the maximum boundary of N(t)}$$

Therefore

$$\Gamma = \{S, E, B, I, R\} \in \mathbb{R}_+{}^5; 0 \leq N(t) \leq \frac{\Lambda}{\mu} \tag{3.9}$$

This shows that the solution of the model equation 3.1 which start at the boundary (Γ) converge to the region and remain bounded at maximum $\{\frac{\Lambda}{\mu}\}$. This implies that the model is well posed and bounded since it deals with human population.

3.4 Model Analysis

In this section, the effective reproduction number, burglary free equilibrium points, burglary endemic equilibrium points and their stabilities are determined.

3.4.1 Effective Reproduction Number (R_e)

The effective reproduction number R_e is the average number of people who become burglars as a result of an entry of one burglar into the human population.[29]. Effective reproduction number is a very important tool for determining the impact of burglary in the susceptible population. It is used to understand the effect or capacity of burglary in the society when there is no any control measures. When there is other measures of reducing burglary such as creation of job opportunities among youths, carrying out intensive patrol by police officers among others, reproduction number will be maintained less than unity. R_e= Spectral radius of the matrix

$$FV^{-1} \tag{3.10}$$

Where F is the Jacobian matrix of $\mathcal{F}$ which is the rate of appearance of new burglars in the compartments. V is the Jacobian of $\mathcal{V}$ which is the rate of transfer of individuals in and out of the compartment. Reproduction number is given by $R_e = FV^{-1}$ and it is calculated by the next generation matrix approach[30]. Using the exposed and burglary equations, R_e is computed as follow;

$$\frac{dE}{dt} = \frac{\delta\tau(1-\omega(B(t)+\lambda E(t))}{N(t)}S + (1-\theta)R - (\alpha+\mu)E$$
$$\frac{dB}{dt} = \alpha E + \theta R - (\rho+\gamma+\mu)B$$

We define $\mathcal{F}$ as

$$\mathcal{F} = \begin{pmatrix} \beta S \\ 0 \end{pmatrix} \tag{3.11}$$

but $\beta = \frac{\delta\tau(1-\omega)(B(t)+\lambda E(t))}{N(t)}$

therefore

$$\mathcal{F} = \begin{pmatrix} \frac{\delta\tau(1-\omega(B(t)+\lambda E(t))S}{N(t)} \\ 0 \end{pmatrix} \tag{3.12}$$

$$\mathcal{V} = \begin{pmatrix} (\alpha+\mu)E - (1-\theta)R \\ (\rho+\gamma+\mu)B - \alpha E - \theta R \end{pmatrix} \tag{3.13}$$

The Jacobian of $\mathcal{F}$ at burglary free equilibrium point denoted as F is given by;

$$\mathbb{F} = \begin{pmatrix} \frac{\partial \mathbf{f_1}}{\partial \mathbf{E}} & \frac{\partial \mathbf{f_1}}{\partial \mathbf{B}} \\ \frac{\partial \mathbf{f_2}}{\partial \mathbf{E}} & \frac{\partial \mathbf{f_2}}{\partial \mathbf{B}} \end{pmatrix} = \begin{pmatrix} \frac{\delta\tau(1-\omega)\lambda S}{N} & \frac{\delta\tau(1-\omega)S}{N} \\ 0 & 0 \end{pmatrix} \tag{3.14}$$

Similarly the Jacobian of $\mathcal{V}$ at burglary free equilibrium denoted as V is given by

$$\mathbb{J}_{\mathbb{V}} = \begin{pmatrix} \dfrac{\partial \mathbf{v_1}}{\partial \mathbf{E}} & \dfrac{\partial \mathbf{v_1}}{\partial \mathbf{B}} \\ \\ \dfrac{\partial \mathbf{v_2}}{\partial \mathbf{E}} & \dfrac{\partial \mathbf{v_2}}{\partial \mathbf{B}} \end{pmatrix} = \begin{pmatrix} \alpha + \mu & 0 \\ \\ -\alpha & \rho + \gamma + \mu \end{pmatrix} \tag{3.15}$$

Now

$$V^{-1} = \frac{1}{(\rho + \gamma + \mu)(\alpha + \mu)} \begin{pmatrix} \rho + \gamma + \mu & 0 \\ \\ \alpha & \alpha + \mu \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\alpha+\mu} & 0 \\ \frac{\alpha}{(\rho+\gamma+\mu)(\alpha+\mu)} & \frac{1}{\rho+\gamma+\mu} \end{pmatrix} \tag{3.16}$$

Therefore

$$FV^{-1} = \begin{pmatrix} \frac{\delta\tau(1-\omega)\lambda S}{N} & \frac{\delta\tau(1-\omega)S}{N} \\ \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\alpha+\mu} & 0 \\ \\ \frac{\alpha}{(\rho+\gamma+\mu)(\alpha+\mu)} & \frac{1}{\rho+\gamma+\mu} \end{pmatrix}$$

$$FV^{-1} = \begin{pmatrix} (\frac{\delta\tau(1-\omega)\lambda S}{(\alpha+\mu)N} + \frac{\delta\tau(1-\omega)\alpha S}{N(\alpha+\mu)(\rho+\gamma+\mu)}) & \frac{\delta\tau(1-\omega)S}{N(\rho+\gamma+\mu)} \\ \\ 0 & 0 \end{pmatrix} \tag{3.17}$$

The largest eigenvalue of FV^{-1} is the effective reproduction number (R_e)

$$\left| \begin{pmatrix} (\frac{\delta\tau(1-\omega)\lambda S}{(\alpha+\mu)N} + \frac{\delta\tau(1-\omega)\alpha S}{N(\alpha+\mu)(\rho+\gamma+\mu)}) - \lambda & \frac{\delta\tau(1-\omega)S}{N(\rho+\gamma+\mu)} \\ \\ 0 & 0 - \lambda \end{pmatrix} \right| = 0$$

$$\lambda^2 - \lambda(\frac{\delta\tau(1-\omega)\lambda S}{(\alpha+\mu)N} + \frac{\delta\tau(1-\omega)\alpha S}{N(\alpha+\mu)(\rho+\gamma+\mu)}) = 0 \tag{3.18}$$

$$\lambda^2 - \lambda(\frac{\delta\tau(1-\omega)\lambda S}{(\alpha+\mu)N} + \frac{\delta\tau(1-\omega)\alpha S}{N(\alpha+\mu)(\rho+\gamma+\mu)}) = 0$$

$$\lambda_1 = 0$$
$$\lambda_2 = (\frac{\delta\tau(1-\omega)\lambda S}{(\alpha+\mu)N} + \frac{\delta\tau(1-\omega)\alpha S}{N(\alpha+\mu)(\rho+\gamma+\mu)}) \tag{3.19}$$

In burglary free population S=$N = \frac{\Lambda}{\mu}$, Therefore the effective reproduction number of model equation 3.1 is defined as

$$R_e = \delta\tau(1-\omega)\{\frac{\lambda(\rho+\gamma+\mu)+\alpha}{(\alpha+\mu)(\rho+\gamma+\mu)}\} \tag{3.20}$$

This determines whether or not burglary will invade the population. Criminologically, it implies that if $R_e < 1$, burglary will die out of the population and when $R_e > 1$, burglary will persist in the population

3.4.2 Existence of Burglary Free Equilibrium point(BFE)

The burglary-free equilibrium BFE, represented by E^0 describes a stable population state completely devoid of burglary activity. This equilibrium condition occurs when all burglary-related compartments vanish, specifically when $E(t) = B(t) = I(t) = R(t) = 0$,. The BFE solution is derived by setting the normalized system (Equation 3.1) to zero and solving under these conditions[27].

$$\begin{aligned} \Lambda - (\frac{\delta\tau(1-\omega)(B(t)+\lambda E(t))}{N} + \mu)S &= 0 \\ (\frac{\delta\tau(1-\omega)(B(t)+\lambda E(t))}{N})S + (1-\theta)R - (\alpha+\mu)E &= 0 \\ \alpha E + \theta R - (\rho+\gamma+\mu)B &= 0 \\ \gamma B - (\phi+\mu)I &= 0 \\ \phi I - (\mu+1)R &= 0 \end{aligned} \tag{3.21}$$

Therefore burglary free-equilibrium of Equation (3.1) is given by
$E^0 = \{S(t), E(t), B(t), I(t), R(t)\} = (\frac{\Lambda}{\mu}, 0, 0, 0, 0)$

3.4.3 Existence of Burglary Endemic Equilibrium Point

The endemic equilibrium point $E^* = (S^*, E^*, B^*, I^*, R^*)$ represents a steady-state solution of the burglary transmission model where criminal activity persists in the population. This equilibrium condition occurs when all compartment values remain positive and stable over time. The solution is obtained by setting the right-hand side of Equation 3.1 equal to zero and solving the resulting system of equations.

$$\begin{aligned}
\Lambda - (\frac{\delta\tau(1-\omega)(B(t)+\lambda E(t))}{N(t)} + \mu)S = 0 \\
\frac{\delta\tau(1-\omega)(B(t)+\lambda E(t))}{N(t)}S + (1-\theta)R - (\alpha+\mu)E = 0 \\
\alpha E + \theta R - (\rho+\gamma+\mu)B = 0 \\
\gamma B - (\phi+\mu)I = 0 \\
\phi I - (\mu+1)R = 0
\end{aligned} \tag{3.22}$$

by backward substitution

$$\begin{aligned}
\phi I &= (\mu+1)R \\
I &= \frac{(\mu+1)R}{\phi}
\end{aligned} \tag{3.23}$$

$\gamma B - (\phi+\mu)I = 0$

$$I = \frac{\gamma B^*}{(\phi+\mu)} \tag{3.24}$$

Solving (3.23) and (3.24) simultaneously

$$R^* = \frac{\phi\gamma B^*}{(\mu+1)(\phi+\mu)} \tag{3.25}$$

$\alpha E + \theta R - (\rho + \gamma + \mu)B = 0$

$$\alpha E = \{(\rho + \gamma + \mu) - \frac{\theta\phi\gamma}{(\theta + \mu)(\phi + \mu)}\}B$$
$$E^* = (\frac{(\rho + \gamma + \mu)(\theta + \mu)(\phi + \mu) - \theta\phi\gamma}{\alpha(\theta + \mu)(\phi + \mu)})B^* \tag{3.26}$$

by substituting E^*, into $\Lambda - (\frac{\delta\tau(1-\omega)(B(t)+\lambda E(t))}{N(t)} + \mu)S$ it leads to

$$S^* = \frac{N((\alpha + \mu)(\rho + \gamma + \mu)(\mu + 1)(\phi + \mu) - (\alpha + \mu)\theta\gamma\phi - (1 - \theta)\alpha\gamma\phi)}{\delta\tau(1 - \omega)((\alpha(\phi + \mu)(\mu + 1) + \lambda(\rho + \gamma + \mu)(\phi + \mu)(\mu + 1) - \lambda\theta\gamma\phi)}$$

S^* in terms or R_e

$$S^* = \frac{N((\alpha + \mu)(\rho + \gamma + \mu)(\mu + 1)(\phi + \mu) - (\alpha + \mu)\theta\gamma\phi - (1 - \theta)\alpha\gamma\phi)}{R_e(\alpha + \mu)(\rho + \gamma + \mu)(\mu + 1)(\phi + \mu) - \delta\tau(1 - \omega)\lambda\theta\gamma\phi} \tag{3.27}$$

Let $x = N((\alpha + \mu)(\rho + \gamma + \mu)(\mu + 1)(\phi + \mu) - (\alpha + \mu)\theta\gamma\phi - (1 - \theta)\alpha\gamma\phi)$

$y = R_e(\alpha + \mu)(\rho + \gamma + \mu)(\mu + 1)(\phi + \mu) - \delta\tau(1 - \omega)\lambda\theta\gamma\phi$

Therefore $S = \frac{x}{y}$

$$\frac{\Lambda}{S} - \mu = \frac{\delta\tau(1 - \omega)B(t)}{N} + \frac{\delta\tau(1 - \omega)\lambda E(t)}{N} \tag{3.28}$$

Substituting equation 3.26 and 3.27 in 3.28, the result obtained is as follows

$$B^* = \frac{N\alpha((\phi + \mu)(\mu + 1)(\Lambda y - \mu x))}{xy} \tag{3.29}$$

Now substituting equation 3.29 in 3.24, 3.25 & 3.26 the results obtained were as follows

$$I^* = \frac{N\alpha\gamma((\phi + \mu)(\mu + 1)(\Lambda y - \mu x))}{(\phi + \mu)xy}$$
$$R^* = \frac{N\alpha\gamma\phi((\phi + \mu)(\mu + 1)(\Lambda y - \mu x))}{(\phi + \mu)(\mu + 1)xy} \tag{3.30}$$
$$E^* = \frac{((\rho + \gamma + \mu)(\mu + 1)(\phi + \mu) - \theta\gamma\phi)(N\alpha\gamma\phi(\mu + 1)(\phi + \mu)(\Lambda y - \mu x))}{\alpha(\phi + \mu)(\mu + 1)xy}$$

therefore

$$
\begin{aligned}
S^* &= \frac{x}{y} \\
E^* &= \frac{((\rho+\gamma+\mu)(\mu+1)(\phi+\mu)-\theta\gamma\phi)(N\alpha\gamma\phi((1-\theta)+\mu)(\phi+\mu)(\Lambda y-\mu x))}{\alpha(\phi+\mu)(\mu+1)xy} \\
B^* &= \frac{N\alpha((\phi+\mu)(\mu+1)(\Lambda y-\mu x))}{xy} \\
I^* &= \frac{N\alpha\gamma((\phi+\mu)(\mu+1)(\Lambda y-\mu x))}{(\phi+\mu)xy} \\
R^* &= \frac{N\alpha\gamma\phi((\phi+\mu)((1-\theta)+\mu)(\Lambda y-\mu x))}{(\phi+\mu)(\mu+1)xy}
\end{aligned}
$$

3.4.4 Local stability of Burglary Free Equilibrium Point

To check for the local stability of burglary free equilibrium point, linearization of Equation (3.1) at BFE is taken into consideration.

Theorem 3.4.1. *The burglary free equilibrium E^0 of the model equation (3.1) is locally asymptotically stable whenever $R_e < 1$ and unstable when $R_e > 1$.*

Proof. Consider the Jacobian matrix of the linearized model equation (1) at BFE given by

$$
= \begin{pmatrix}
-\frac{\delta\tau(1-\omega)[B(t)+\lambda E(t)}{N}-\mu & -\frac{\delta\tau(1-\omega)\lambda S}{N} & -\frac{\delta\tau(1-\omega)S}{N} & 0 & 0 \\
\frac{\delta\tau(1-\omega)[B(t)+\lambda E(t)]}{N} & \frac{\delta\tau(1-\omega)\lambda S}{N}-(\alpha+\mu) & \frac{\delta\tau(1-\omega)S}{N} & 0 & (1-\theta) \\
0 & \alpha & -(\rho+\gamma+\mu) & 0 & \theta \\
0 & 0 & \gamma & -(\phi+\mu) & 0 \\
0 & 0 & 0 & \phi & -(\mu+1)
\end{pmatrix} \tag{3.31}
$$

but at (BFE) $B = E = 0$ and $N = S$ the Jacobian matrix is reduced to.

$$= \begin{pmatrix} -\mu & -\delta\tau(1-\omega)\lambda & -\delta\tau(1-\omega) & 0 & 0 \\ 0 & \delta\tau(1-\omega)\lambda-(\alpha+\mu) & \delta\tau(1-\omega) & 0 & (1-\theta) \\ 0 & \alpha & -(\rho+\gamma+\mu) & 0 & \theta \\ 0 & 0 & \gamma & -(\phi+\mu) & 0 \\ 0 & 0 & 0 & \phi & -(\mu+1) \end{pmatrix} \quad (3.32)$$

The first eigenvalue $\lambda_1 = -\mu < 0$ by observation and the Jacobian matrix at BFE is reduced to;

$$J_{BFE} = \begin{pmatrix} \delta\tau(1-\omega)\lambda-(\alpha+\mu) & \delta\tau(1-\omega) & 0 & (1-\theta) \\ \alpha & -(\rho+\gamma+\mu) & 0 & \theta \\ 0 & \gamma & -(\phi+\mu) & 0 \\ 0 & 0 & \phi & -(\mu+1) \end{pmatrix} \quad (3.33)$$

To analyze the stability of the above remaining Jacobian matrix at BFE , we compute the trace and determinant.

Theorem 3.4.2. ***Trace-determinant criterion*** *If the trace($J_E{}^o < 0$) and det($J_E{}^o > 0$), then the burglary free equilibrium point is locally asymptotically stable whenever $R_e < 1$.*

Proof. Let the trace be denoted by ϖ therefore at BFE, it is given by the sum of the main diagonal

$$\varpi(J_E{}^0) = (\delta\tau(1-\omega)\lambda - (\alpha+\mu)) - (\rho+\gamma+\mu) - (\phi+\mu) - (\mu+1) \quad (3.34)$$

Since all model parameters are positive, the trace of the above Jacobian matrix remain negative provided that $\delta\tau(1-\omega)\lambda \leq (\alpha+\mu)$. Reproduction number can be simplifed as;

$$R_e(\alpha+\mu)(\rho+\gamma+\mu) - \frac{\delta\tau(1-\omega)\alpha}{(\rho+\gamma+\mu)} = \delta\tau(1-\omega)\lambda \quad (3.35)$$

Substituting equation 3.37 into 3.36, it yield;

$$\varpi(J_E{}^0) = (R_e - 1)(\alpha + \mu) - \frac{\delta\tau(1-\omega)\alpha}{(\rho+\gamma+\mu)} - (\rho + \gamma + 3\mu + \phi + 1) \tag{3.36}$$

□

The trace remains negative whenever ($R_e < 1$), this implies that all the eigenvalues are negative real parts. The determinant of the above Jacobian matrix is determined by the use of block triangle method where is it partitioned into four sub-matrices as shown below;

$$J_{E^0} = \begin{pmatrix} A & B \\ 0 & D \end{pmatrix},$$

where

$$A = \begin{pmatrix} \delta\tau(1-\omega)\lambda - (\alpha+\mu) & \delta\tau(1-\omega) \\ \alpha & -(\rho+\gamma+\mu) \end{pmatrix}, \quad B = \begin{pmatrix} 0 & (1-\theta) \\ 0 & \theta \end{pmatrix},$$

$$D = \begin{pmatrix} -(\phi+\mu) & 0 \\ \phi & -(\mu+1) \end{pmatrix}.$$

By the use the use of block determinant formula,

$$\det(J_{E^0}) = \det(A) \cdot \det(D). \tag{3.37}$$

$$\det(A) = (\delta\tau(1-\omega)\lambda - (\alpha+\mu) - (\rho+\gamma+\mu) - (\delta\tau(1-\omega)\alpha.$$

$$\det(D) = (-(\phi+\mu)(-(\mu+1) - (0\cdot\phi) = (\phi+\mu)(\mu+1). \tag{3.38}$$

The product of $\det(A)$ and $\det(D)$ yields

$$\det(J_{E^0}) = \left[(\delta\tau(1-\omega)\lambda - (\alpha+\mu))\left(-(\rho+\gamma+\mu)\right) - (\delta\tau(1-\omega)\alpha\right](\phi+\mu)(\mu+1).$$

By substituting the vale of R_e in the above equation;

$$\det(J_{E^0}) = ((1-R_e)(\alpha+\mu)(\rho+\gamma+\mu)(\phi+\mu)(\mu+1)) \tag{3.39}$$

The determinant remains positive provided that $R_e < 1$ and all model parameters are positive. The determinant conditions $\det(J_{E^0}) > 0$ hold, and the Routh-Hurwitz criterion is satisfied when $\mathcal{R}_e < 1$, the Burglary-Free Equilibrium is locally asymptotically stable for $\mathcal{R}_e < 1$.

Theorem 3.4.1 implies that given a small perturbation of BFE the solution of model equation (1) will eventually converge to BFE whenever $R_e < 1$. Criminologically, it implies that a small influx of active burglars into susceptible population will not trigger a significant outbreak of burglary in the society. When $R_e < 1$, the spread of burglary will inevitably decline if the initial number of active burglars lies within the basin of attraction of burglary free equilibrium (BFE). Under these conditions, any small influx of burglars will result in the model solution converging to (BFE). This implies that as long as $R_e < 1$, burglary practices will not proliferate within the population. Consequently, crime will decline and burglary will not develop in the population. To ensure that burglary elimination is independent in the susceptible population, it is necessary to show that it is globally asymptotically stable. □

3.4.5 Global stability of Burglary Free Equilibrium

The global asymptotic stability of the burglary-free equilibrium for system (3.1) can be analyzed through Lyapunov's direct method. When the control reproduction number satisfies $R_e < 1$, the equilibrium E_0 is globally asymptotically stable

Theorem 3.4.3. *Let $E_0 = (S, E, B, I, R)$ be the burglary free equilibrium of the system;*

$\frac{dX}{dt} = f(X), X = (S, E, B, I, R)$

Suppose there exist a linear Lyapunov function of the form $P = y_1 E + y_2 B$ where y_1,$y_2 > 0$ are constant and the derivative of the function P along the trajectories satisfies; $\frac{dP}{dt} = y_1 \frac{dE}{dt} + y_2 \frac{dB}{dt} \leq 0$ for all (E,B) then E_0 is globally asymptotically *stable when $R_e < 0$.*

Proof. Consider the Lyapunov function [20] below

$$P = y_1 E + y_2 B \tag{3.40}$$

Taking derivative with respect to (t), we have

$$\frac{dP}{dt} = y_1 \frac{dE}{dt} + y_2 \frac{dB}{dt} \tag{3.41}$$

Substituting the value of $\frac{dE}{dt}$ and $\frac{dB}{dt}$ from the model equation 3.1 in the above equation

$$\frac{dP}{dt} = y_1 \frac{\delta\tau(1-\omega)B(t)S}{N} + y_1 \frac{\delta\tau(1-\omega)\lambda E(t)S}{N} + y_1(1-\theta)R - y_1(\alpha+\mu)E + y_2\alpha E + y_2\theta R - y_2(\rho+\gamma+\mu)B \tag{3.42}$$

Taking the coefficient of E(t) to express y_2 in terms of y_1

$y_1\delta\tau(1-\omega)\lambda E(t) - y_1(\alpha+\mu)E(t) + y_2\alpha E(t)$.

Now

$y_2 = -y_1(\frac{\delta\tau(1-\omega)\lambda-(\alpha+\mu)}{\alpha})$ and by letting $y_1 = 1$

At the burglary free equilibrium point $S = N = \frac{\Lambda}{\mu}$, Therefore

$$\frac{dP}{dt} \leq y_1\delta\tau(1-\omega)B(t) - y_2(\rho+\gamma+\mu)B(t) \tag{3.43}$$

by substituting the value of y_2 in equation (3.43) it leads to the following

$$\frac{dP}{dt} \leq (\delta\tau(1-\omega) + (\rho+\gamma+\mu)(\frac{\delta\tau(1-\omega)\lambda-(\alpha+\mu)}{\alpha}))B(t) \tag{3.44}$$

$$\frac{dP}{dt} \leq \frac{(\rho+\gamma+\mu)(\alpha+\mu)}{\alpha}(\frac{\delta\tau(1-\omega)\alpha}{\rho+\gamma+\mu)(\alpha+\mu)} + \frac{\delta\tau(1-\omega)\lambda}{\alpha+\mu} - 1)B(t)$$

but $R_e = \frac{\delta\tau(1-\omega)\alpha}{\rho+\gamma+\mu)(\alpha+\mu)} + \frac{\delta\tau(1-\omega)\lambda}{\alpha+\mu}$, by substituting R_e in equation (3.44)

$$\frac{dP}{dt} \leq \frac{(\rho+\gamma+\mu)(\alpha+\mu)}{\alpha}((R_e - 1))B(t) \tag{3.45}$$

□

Hence from the proof, it is clear that $\frac{dP}{dt} \leq 0$ if $R_e < 1$ and $\frac{dP}{dt} = 0$ if $B = 0$. Therefore, the solution of Equation (3.1) satisfies $B \longrightarrow 0$ as $t \longrightarrow \infty$ by LaSalle's principle [20] thus the Burglary free equilibrium point is globally asymptotically stable whenever $R_e < 1$. The criminological implication of this result shows that any small influx of burglars in the population will not increase the number of burglars thus the burglary will die out in the population or it can be eliminated from the susceptible population if R_e is maintained to a value less than unity.

3.4.6 Local Stability of Burglary Endemic Equilibrium point

For burglary to be endemic in a susceptible population $E^* > 0$, whenever $R_e > 1$. At this point burglary persistence occurs in the population thus $B > 0$.

Consider;

$B^* = \frac{N\alpha((\phi+\mu)(\mu+1)(\Lambda y-\mu x))}{xy}$

As $t \longrightarrow \infty$ and $N \longrightarrow \frac{\Lambda}{\mu}$, then B^* becomes

$$B^* = \frac{\Lambda\alpha((\phi+\mu)(\mu+1)(\Lambda y-\mu x))}{\mu xy} > 0$$

where $x = N((\alpha+\mu)(\rho+\gamma+\mu)(\mu+1)(\phi+\mu) - (\alpha+\mu)\theta\gamma\phi - (1-\theta)\alpha\gamma\phi)$

$y = R_e(\alpha+\mu)(\rho+\gamma+\mu)(\mu+1)(\phi+\mu) - \delta\tau(1-\omega)\lambda\theta\gamma\phi$

This inequality holds if and only if $R_e > 1$, thus $B^*(t) > 0$

The endemic equilibrium point of the Equation (3.1) is locally asymptotically stable in Γ if $R_e > 1$ otherwise unstable.

Consider the Jacobian matrix of linearized model equation 3.1, evaluated at endemic equilibrium point, is given by the following

$$J_E{}^* = \begin{pmatrix} -T_1-\mu & -T_3 & -T_3 & 0 & 0 \\ T_1 & T_2 & T_3 & 0 & (1-\theta) \\ 0 & \alpha & -(\rho+\gamma+\mu) & 0 & \theta \\ 0 & 0 & \gamma & -(\phi+\mu) & 0 \\ 0 & 0 & 0 & \phi & -(\mu+1) \end{pmatrix} \tag{3.46}$$

Where $T_1 = \frac{(\delta\tau(1-\omega)(B^*+\lambda E^*))}{N}$, $T_2 = \frac{\delta\tau(1-\omega)\lambda S^*}{N} - (\alpha+\mu)$ and $T_3 = \frac{\delta\tau(1-\omega)\lambda S^*}{N}$

By the Routh-Hurwitz criterion, the necessary and sufficient condition for all the roots of the characteristic polynomial with real coefficients to have negative real parts is that all its eigenvalues are negative[17]. This occurs if the trace the Jacobin matrix is negative and the determinant is positive

From the above Jacobian matrix linearized at E^*, the condition for the trace to be negative is

$\frac{\delta\tau(1-\omega)\lambda S^*}{N} < (\alpha+\mu)$

The determinant is give by

$$DetJ_{E^*} = T_2(T_1+\mu)(\rho+\gamma+\mu)(\phi+\mu)(\mu+1) + \alpha T_3(T_1+\mu)(\phi+\mu)(\mu+1) - T_1T_3(\phi+\mu)(\mu+1)((\rho+\gamma+\mu)+\alpha) - T_2(T_1+\mu)\phi\gamma\theta \tag{3.47}$$

When B^*, E^* and S^* are substituted in T_1, T_2, T_3 where

$$S^* = \frac{Nk_1(k_2k_3k_4 - k_1\theta\gamma\phi - k_5\alpha\theta\gamma\phi)}{R_ek_1k_2k_3k_4 - k_6\lambda\theta\gamma\phi}$$

$$B^* = \frac{N\alpha k_3k_4((\Lambda R_ek_1k_2k_3k_4 - \Lambda k_6\lambda\phi\gamma\theta) - \mu Nk_1(k_2k_3k_4 - k_1\phi\gamma\theta - k_5\alpha\phi\gamma\theta))}{(Nk_1(k_2k_3k_4 - k_1\phi\gamma\theta - k_5\alpha\phi\gamma\theta))(R_ek_1k_2k_3k_4 - k_6\lambda\phi\gamma\theta))}$$

$$E^* = \frac{((k_2k_3k_4 - \phi\gamma\theta)(N\alpha\gamma\phi k_3k_4(\Lambda R_ek_1k_2k_3k_4 - \Lambda k_6\lambda\phi\gamma\theta) - \mu Nk_1(k_2k_3k_4 - k_1\phi\gamma\theta - k_5\alpha\phi\gamma\theta))}{\alpha k_3k_4((Nk_1(k_2k_3k_4 - k_1\phi\gamma\theta - k_5\alpha\phi\gamma\theta))(R_ek_1k_2k_3k_4 - k_6\lambda\phi\gamma\theta))}$$

and

$k_1 = (\alpha+\mu), k_2 = (\rho+\gamma+\mu), k_3 = (\phi+\mu), k_4 = (\mu+1), k_5 = (1-\theta), k_6 = \delta\tau(1-\omega)$

The determinant remains positive provided that all model parameters are positive and $R_e > 1$. Thus E^* is locally asymptotically stable whenever $R_e > 1$ by Routh-Hurwitz. The criminological implication of this results is that a small influx of active burglars into susceptible population will leads to an outbreak of burglary in the society. When $R_e > 1$, the spread of burglary will grow exponentially in the population. This implies that any small influx of burglars will influence other individuals to burglary practices.

3.4.7 Global Stability of Burglary Endemic Equilibrium Point

The global stability of burglary endemic equilibrium is obtained by the use of Lyapunov method and LaSalle Invariance principle[3].

Theorem 3.4.4. *Let $(S^*, E^*, B^*, I^*, R^*)$ be the endemic equilibrium of burglary dynamic system;*

$$\frac{dX}{dt} = f(X), X = (S^*, E^*, B^*, I^*, R^*)$$

Suppose there exist a continuously differentiable Lyapunov function $U(X)$ such that the derivative of the system along the trajectories satisfy $U^{'}(X) \leq 0$ then, the endemic burglary equilibrium is globally asymptotically stale

Proof. Consider non-linear Lyapunov function.

$$U = m_1(S - S^* - S^* \ln \frac{S}{S^*}) + m_2(E - E^* - E^* \ln \frac{E}{E^*}) + m_3(B - B^* - B^* \ln \frac{B}{B^*}) + m_4(I - I^* - I^* \ln \frac{I}{I^*}) + m_5(R - R^* - R^* \ln \frac{R}{R^*}) \quad (3.49)$$

Where U is in the interior region Γ. E^* is the global minimum of U on Γ and $U : \{S(t), E(t), B(t), I(t), R(t)\} = 0$

The time derivative of U along the solution of model equation 3.1 is given by

$$U^1 = m_1(1 - \frac{S^*}{S})S' + m_2(1 - \frac{E^*}{E})E' + m_3(1 - \frac{B^*}{B})B' + m_4(1 - \frac{I^*}{I})I' + m_5(1 - \frac{R^*}{R})R'$$

Replacing S', E', B', I', R' from equation 3.1, we obtain

$$U^1 = m_1(1-\frac{S^*}{S})(\Lambda-(\beta+\mu)S)+m_2(1-\frac{E^*}{E})(\beta S+(1-\theta)R-(\alpha+\mu)E)+m_3(1-\frac{B^*}{B})$$
$$(\alpha E+\theta R-(\rho+\gamma+\mu)B)+m_4(1-\frac{I^*}{I}(\gamma B-(\phi+\mu)I)'+m_5(1-\frac{R^*}{R})(\phi I-(\mu+1)R) \tag{3.50}$$

At endemic taking E^* into account and $m_i > 0$ for $i = 1,2,3,4,5$ in Γ and its constants m_i are continuously differentiable in Γ and $U(\Gamma^*) = 0$, where $\Gamma = (S^*, E^*, B^*, I^*, R^*)$. Global stability for endemic hold if $U' \leq 0$. Therefore at endemic equilibrium U' is given as follows

$$\begin{aligned}U' = & m_1(1-\frac{S^*}{S})\,((\beta+\mu)S^*-(\beta+\mu)S)+m_2(1-\frac{E^*}{E})((\alpha+\mu)E^*-(\alpha+\mu)E)\\ &+m_3(1-\frac{B^*}{B})((\rho+\gamma+\mu)B^*-(\rho+\gamma+\mu)B)+m_4(1-\frac{I^*}{I})((\phi+\mu)I^*-(\phi+\mu)I)\\ &+m_5(1-\frac{R^*}{R})(\mu+1)R^*-((1-\theta)+\mu)R)\end{aligned} \tag{3.51}$$

By rearranging the terms in equation 3.51 leads to the following

$$\begin{aligned}U' =& m_1(1-\frac{S^*}{S})(-(\beta+\mu)(S-S^*))+m_2(1-\frac{E^*}{E})(-(\alpha+\mu)(E-E^*))+\\ & m_3(1-\frac{B^*}{B})(-(\rho+\gamma+\mu)(B-B^*))m_4(1-\frac{I^*}{I})(-(\phi+\mu)(I-I^*))+\\ & m_5(1-\frac{R^*}{R})(-(\mu+1)(R-R^*))\end{aligned} \tag{3.52}$$

By factorization of terms in equation 3.52 leads to the following

$$\begin{aligned}U' =& m_1(\frac{S-S^*}{S})(-(\beta+\mu)S(1-\frac{S^*}{S}))+m_2(\frac{E-E^*}{E})(-(\alpha+\mu)E(1-\frac{E^*}{E}))+\\ & m_3(\frac{B-B^*}{B})(-(\rho+\gamma+\mu)B)(1-\frac{B^*}{B}))+m_4(\frac{I-I^*}{I})(-(\phi+\mu)I(1-\frac{I^*}{I}))+\\ & m_5(\frac{R-R^*}{R})(-(\mu+1)R(1-\frac{R^*}{R}))\end{aligned} \tag{3.53}$$

Therefore

$$\begin{aligned}U' =& m_1(\frac{S-S^*}{S})(-(\beta+\mu)S(\frac{S-S^*}{S}))+m_2(\frac{E-E^*}{E})(-(\alpha+\mu)E(\frac{E-E^*}{E}))+\\ & m_3(\frac{B-B^*}{B})((-(\rho+\gamma+\mu)B(\frac{B-B^*}{B}))+m_4(\frac{I-I^*}{I})(-(\phi+\mu)I(\frac{I-I^*}{I}))+\\ & m_5(\frac{R-R^*}{R})(-(\mu+1)R(\frac{R-R^*}{R}))\end{aligned} \tag{3.54}$$

By opening the bracket

$$U' = -m_1(\beta+\mu)S(\frac{S-S^*}{S})^2 - m_2(\alpha+\mu)E(\frac{E-E^*}{E})^2 - m_3(\rho+\gamma+\mu)B(\frac{B-B^*}{B})^2$$
$$- m_4\phi+\mu)I(\frac{I-I^*}{I})^2 - m_5(\mu+1)R(\frac{R-R^*}{R})$$

From the previous analysis, we noted that all parameters are positive and all state variables $S, E, B, I, R \geq 0$. We also note that $(\frac{S-S^*}{S})^2 \geq 0, (\frac{E-E^*}{E})^2 \geq 0, \frac{B-B^*}{B})^2 \geq 0, (\frac{I-I^*}{I})^2 \geq 0, (\frac{R-R^*}{R})^2 \geq 0$ Hence

$$U' = -\, m_1(\beta+\mu)S(\frac{S-S^*}{S})^2 - m_2(\alpha+\mu)E(\frac{E-E^*}{E})^2 - m_3(\rho+\gamma+\mu)B(\frac{B-B^*}{B})^2$$
$$- m_4\phi+\mu)I(\frac{I-I^*}{I})^2 - m_5(\mu+1)R(\frac{R-R^*}{R})^2 \leq 0 \tag{3.55}$$

□

Since all parameters are positive, it is clear that $U' \leq 0$ and $U' = 0$ when $S = S^*, E = E^*, B = B^*, I = I^*,\ R = R^*$. This implies that the largest compact invariant set in $S(t), E(t), B(t), I(t), R(t) \in \Gamma$: $U = 0$ is a singleton E^*, where E^* is the endemic equilibrium point. Hence, E^* is globally asymptotically stable in the interior of Γ. Global asymptotic stability indicates that, regardless of the initial conditions the system's trajectories will converge to E^* as time progress whenever $R_e > 1$ [16]. This means that any perturbation in the system caused by the introduction of additional burglars in the susceptible population will not destabilize the equilibrium and the system will return to the endemic state E^*. From criminological perspective, when $R_e > 1$ any perturbation of the model by a small influx of burglars into susceptible population influences an average of more than one individual to engage in burglary. This leads to increase rate of burglary in the population. Without effective intervention on unemployment rate, burglary level will persist in the society. For instance, if $R_e = 4$, implies that one active burglar in the susceptible population will influence an average of four other individuals during their interaction. In this case, burglary will grow exponentially in the population until government intervention strategies on unemployment rate are implemented to curb the spread of burglary in the society.

3.4.8 Sensitivity Analysis

Sensitivity analysis is the process of investigating the parameter that have a greater impact on effective reproduction number R_e. This process helps to know the criteria to curb burglary problem in susceptible population as directed by that parameter which has a greater impact. We use Normalized forward sensitivity index to know the elasticity [15].

Table 3.2: **Sensitivity indices of R_e with respect to the model parameters**

Parameters	**Description**	**Values**	**Sensitivity Index**	**Sources**
δ	Rate at which individual become burglar upon contact	0.95week^{-1}	1	[15]
τ	contact rate	0.95 week^{-1}	1	Assumed
λ	Exposure influence rate	0.4week^{-1}	0.7508	
ω	Employment rate	0.4week^{-1}	-0.6667	Assumed
α	Rate of transfer from exposed to burglary	0.15week^{-1}	-.4806	Assumed
μ	Natural death rate	0.05	-0.2776	[15]
γ	Incarceration rate	0.4week^{-1}	-0.2205	Assumed
ρ	Burglary specific death rate	0.002week^{-1}	-0.0011	Assumed

$$Y_x^{R_e} = \frac{x}{R_e} . \frac{\partial R_e}{\partial x} \tag{3.56}$$

where x represent the basic parameters defining effective reproduction number. The effective reproduction number depends on eight parameters which are used to derive analytic expression for each parameter.By carrying out partial derivative of effective reproduction number with respect to all parameters $\delta, \tau, \omega, \lambda, \rho, \gamma, \mu, \alpha$ defining R_e, and applying Normalized sensitivity index formula. The elasticities for quantities of interest are computed as follows;

$$R_e = \frac{\delta\tau(1-\omega)(\lambda(\rho+\gamma+\mu)+\alpha)}{(\alpha+\mu)(\rho+\gamma+\mu)}$$

By the use of quotient rule $R_e = \frac{f(x)}{g(x)} = \frac{f^1(x)g(x)-f(x)g^1(x)}{(g(x))^2}$

Now differentiating R_e with respect to δ

$$f_\delta^1(x) = \tau(1-\omega)(\lambda(\rho+\gamma+\mu)+\alpha)$$
$$g_\delta^1(x) = 0$$

Substituting the values in the elasticity $Y_x^{R_e}$ the result obtained as follows

$$Y_\delta^{R_e} = \frac{\delta(\alpha+\mu)(\rho+\gamma+\mu)}{\delta\tau(1-\omega)[\lambda(\rho+\gamma+\mu)+\alpha]}.(\frac{\tau(1-\omega)[\lambda(\rho+\gamma+\mu)+\alpha][(\alpha+\mu)(\rho+\gamma+\mu)]}{[(\alpha+\mu)(\rho+\gamma+\mu)]^2}) = 1 \tag{3.57}$$

Similarly when the same procedure is applied in other elasticities, the results obtained were as follows.

$$\begin{aligned}
Y_\tau^{R_e} &= \frac{\tau}{R_e}.\frac{\partial R_e}{\partial \tau} = 1\\
Y_\omega^{R_e} &= (\frac{-\omega}{1-\omega})\\
Y_\lambda^{R_e} &= \frac{(\rho+\gamma+\mu)}{(\rho+\gamma+\mu)+\alpha}\\
Y_\rho^{R_e} &= (\frac{\rho}{(\rho+\gamma+\mu)+\alpha} - \frac{\rho}{(\rho+\gamma+\mu)})\\
Y_\gamma^{R_e} &= (\frac{\gamma}{(\rho+\gamma+\mu)+\alpha} - \frac{\gamma}{(\rho+\gamma+\mu)})\\
Y_\mu^{R_e} &= (\frac{\mu}{(\rho+\gamma+\mu)+\alpha} - \frac{2\mu^2+\mu(\rho+\gamma+\alpha)}{(\alpha+\mu)(\rho+\gamma+\mu)})\\
Y_\alpha^{R_e} &= (\frac{\alpha}{\lambda(\rho+\gamma+\mu)+\alpha} - \frac{\alpha}{(\alpha+\mu)})
\end{aligned} \tag{3.58}$$

From the derivative of effective reproduction number with respect to parameters used in defining R_e [15], we can conclude that some sensitivity values of the parameters defining R_e are positive while other are negative. Analytically, it indicates that all values of R_e for sensitivity index that are negative are the most important factors to consider in controlling burglary in a population. Therefore increasing the value of such parameters become the most control criteria of burglary. From table 3.2 the employment rate is the most negative sensitivity parameter to be considered. This shows that a decrease in the value of ω will subsequently increases the number of unemployed individuals hence increases burglary practices in Kenya. Therefore Kenyan government and the society at large should come up with a suitable strategies on how to increase ω so as to alleviate burglary prevalence in the community.

From the table 3.2, all positive indices indicates an increase in the effective reproduction number. This implies that increase in percentage parameter increases R_e hence burglary prevalence.The most positive index is δ, which implies that probability at which susceptible individual become a burglar increases upon contact with the burglar individual[15]. We can deduce that, number of burglars can be reduced from the susceptible population by increasing ω and also increasing the incarceration rate.

CHAPTER FOUR

NUMERICAL SIMULATION

4.1 Introduction

In this section, Equation (3.1) is simulated using the parameters values in the table below. It is done by considering the sensitivity to the parameters in control of burglary in the susceptible population. MATLAB software is used in simulation to show the validity of the analytical part studied in previous sections graphically. This aid to asses the impact of unemployment in the dynamics of burglary in Kenya.

Table 4.1: **Parameter's values**

Parameters	**description**	**values**	**sources**
$S(t)$	Susceptible class	1000	Estimate
$E(t)$	Exposed class	700	Estimate
$B(t)$	Burglar class	200	Estimate
$I(t)$	Incarcerated class	160	Estimate
$R(t)$	Released class	45	Estimate
Λ	Recruitment rate	50	Estimate
θ	Recidivism rate	0.9	[15]
$(1-\theta)$	Rate at which released relapse to exposed class	0.75	[15]
δ	Rate at which individual become burglar upon contact	0.95	[15]
τ	contact rate	0.95	Assumed
λ	Exposure influence rate	0.9	assumed
α	Rate of transfer from exposed to burglary	0.15	Assumed
ω	Employment rate	0.4	[26]
$(1-\omega)$	Unemployment rate	0.6	[26]
ρ	Burglary associated death rate	0.002	Assumed
μ	Natural death rate	0.05	[15]
γ	Incarceration rate	0.4	Assumed
ϕ	Removal rate from prion	0.5	[16]

4.2 Simulation Results and Discussion

In this section, simulation results are presented using the burglary dynamics model as described in the $Figure(4.1)$, $Figure(4.2)$, $Figure(4.3)$ and $Figure(4.4)$. The simulation results focus on the factor influencing the persistence and the spread of burglary in the population. A study done by [10] investigated crime dynamics in developing counties and found that unemployment, and economic instability contribute to persistence of crimes which is applicable to burglary incidents in Kenya. This is lack of employment opportunities, weak policing structures and social inequalities which create an environment where criminal activities thrive. The findings shows that during economic decline, burglary tend to rise due to lack of job opportunities. This implies that without a proper interventions on employment rate in Kenya, burglary persist is the population. These results are confirmed by burglary dynamics as shown in the figures 4.1 and 4.2. The impact of employment rate is also confirmed in figure 4.3 and 4.4

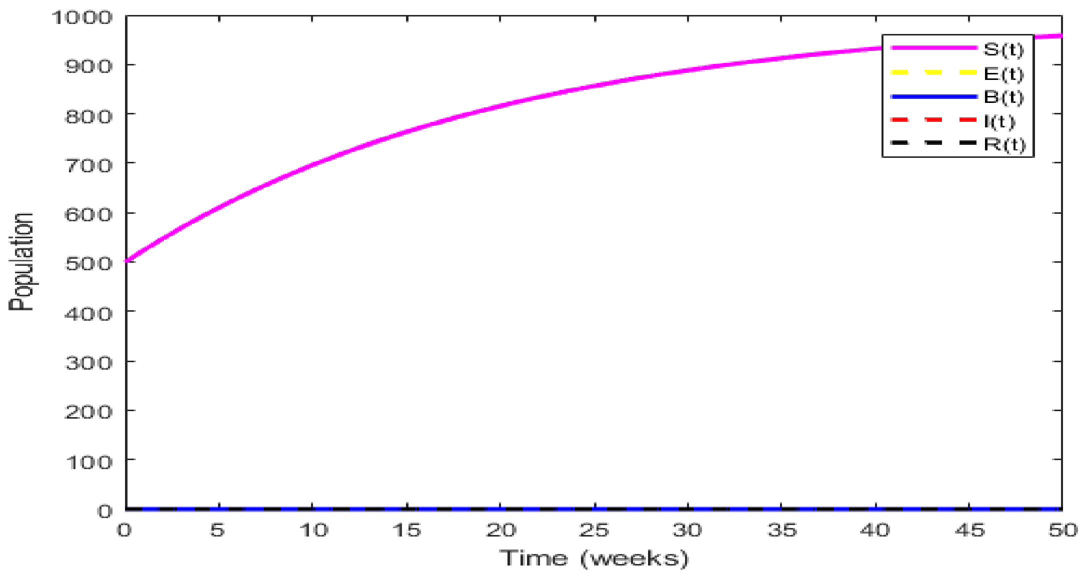

Figure 4.1: Simulation results showing the population when $R_e = 0.5287$ with other parameters made constant

Figure 4.1 shows that the model approaches burglary free equilibrium point where $R_e < 1$. Susceptible population increases exponentially and later stabilizes at $\frac{\Lambda}{\mu}$, this is because the population is free from burglary, that is there is no any susceptible individuals who is exposed to burglary hence no transition of individuals from susceptible class. This leads to constant increase of susceptible population, that later stabilizes when recruitment Λ and natural death μ balances, that is $(S \longrightarrow \frac{\Lambda}{\mu})$.

Burglary remains at zero throughout the entire simulation. This is because there is no transition to burglary class.This implies that, over time, the model reaches a state where there is no emergence of new burglars or active burglars in the population.

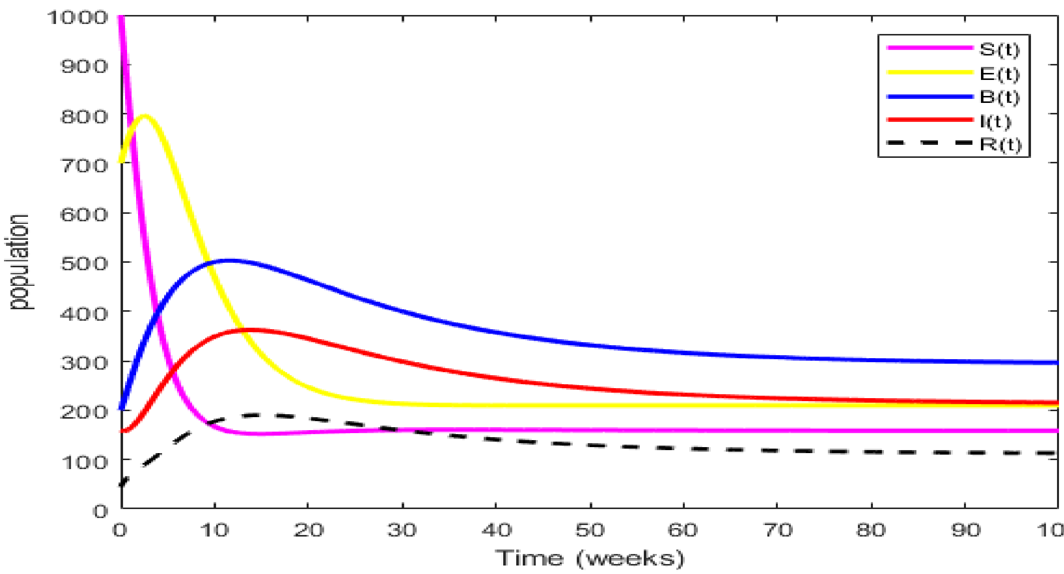

Figure 4.2: Figure shows burglary dynamics in the population when $R_e = 3.1724$ with other parameters made constant

The graph of figure 4.2 shows the burglary persistence in the population when $R_e > 1$. Initially , susceptible individuals decreases rapidly due to a high contact rate τ between susceptible individuals and burglars, a significant probability δ of transition to burglary and a strong influence factor λ. Over time, the susceptible population stabilizes at a lower level as recruitment Λ and outflows from susceptible class reach equilibrium. This suggests that only few individuals remain susceptible in long run.

At the beginning $t = 4$, the number of exposed individuals rises sharply due to a substantial transition from susceptible class. However, after $t = 4$, the exposed population declines asymptotically as most individuals either become burglars at rate α, influenced by low employment rate ω or exit the system through natural death μ. By $t = 30$, the exposed class stabilizes [26]. The burglary population initially grows, reaching a peak at $t = 12$, before decline due to increased incarceration rate γ. However, it eventually stabilizes at a higher level that exposed class, reflecting the persistent nature of burglary in the population. This persistence is driven by a high recruitment rate α, largely influenced by high unemployment rate $(1 - \omega)$. The results indicate that a considerable number of individuals engage in burglary has a direct consequence of economic factor [28].

The incarcerated population starts at a lower level and gradually rises, mirroring the trend of burglary class. This increase is due to the accumulation of burglars before incarceration and the fact that the recruitment rate into incarceration γ is relatively low compared to burglary recruitment rate β. After $t = 12$, the incarcerated population declines as individuals are released back into the society. Over time, it stabilizes when recruitment rate balances the outflows and natural death rate. Released class increases gradually as a results of recruitment ϕ. After $t = 12$, the population declines steadily, as some individuals relapse into burglary direct at the rate θ, a phenomenon known as recidivism, while others transits back to exposed class at the rate $1 - \theta$. This is because some people find it difficult to survive in the society after finishing their jail term and again due to lack of either formal or informal employment.

4.3 Effect of Unemployment Against Burglary Population

In this part , the model is simulated to check the effect of employment rate (ω) against burglary population. There is initial rise in burglary population at different rate for all employment rates, but the rise is but it is much smaller at $\omega = 0.9$.

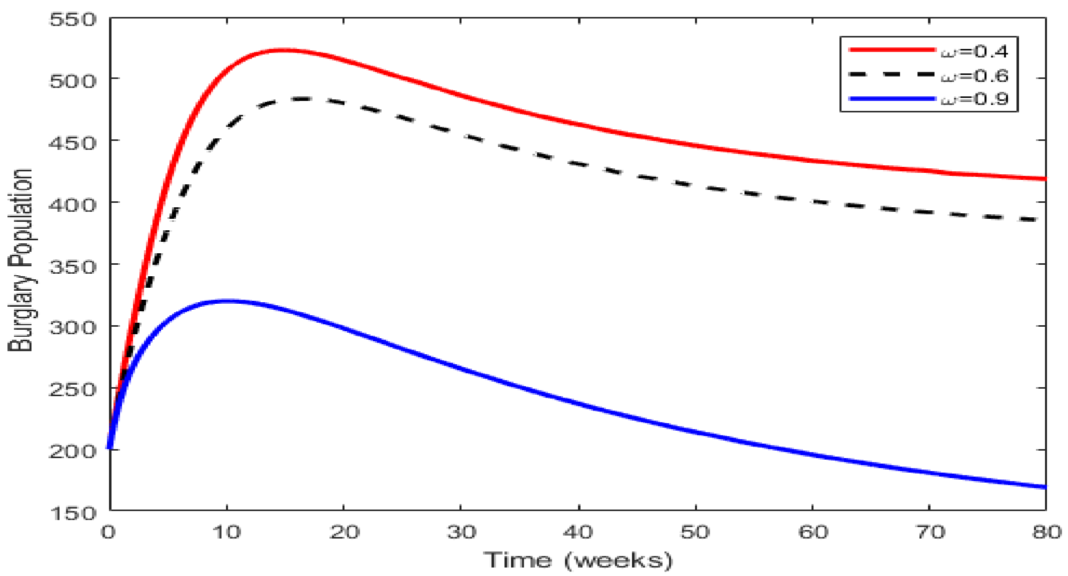

Figure 4.3: Simulation results showing the number of burglars with the variation of employment rate

Figure 4.3 shows the varied employment rate. When the employment rate of 0.4, unemployment rate is 0.6, which is relatively high compared to when employment rate is at 0.6 or 0.9. From the graph, we can note that the lower the rate of employment, the higher the rate of burglary in the community[17]. This indicates that most people revolve in property crime such as burglary when the wages are too low or when there is no employment completely in an economy[27]. this suggest that, there is direct link between unemployment rate and burglary. When unemployment rate declines from $0.6 \longrightarrow 0.4 \longrightarrow 0.1$ or when the employment rate increases from $0.4 \longrightarrow 0.6 \longrightarrow 0.9$, the rate of burglary also decline. This implies that government should create more job opportunities to young people in the society since most burglary incidents are omitted by youths. Established individuals should also empower disorganized sluggards and graduates so as to help in alleviating this burglary problem in the society. From the graph, when $\omega = 0.9$, burglary decline and after longer period of time, it tends to a value close to zero. This indicates that when people's wages increases, they are more likely not to engaged in burglary but when people's wages decline, they are more likely to revolve in burglary since they will have intention of making money or looking for chances of survival even if it is illegitimate.

4.4 Effect of Unemployment Against Incarcerated

In this part the model is simulated to check the effect of employment with the parameter (ω) against Incarcerated population. There is initial rise in the population but at different rate for all employment rates, but it is much smaller when employment rate $\omega = 0.9$.

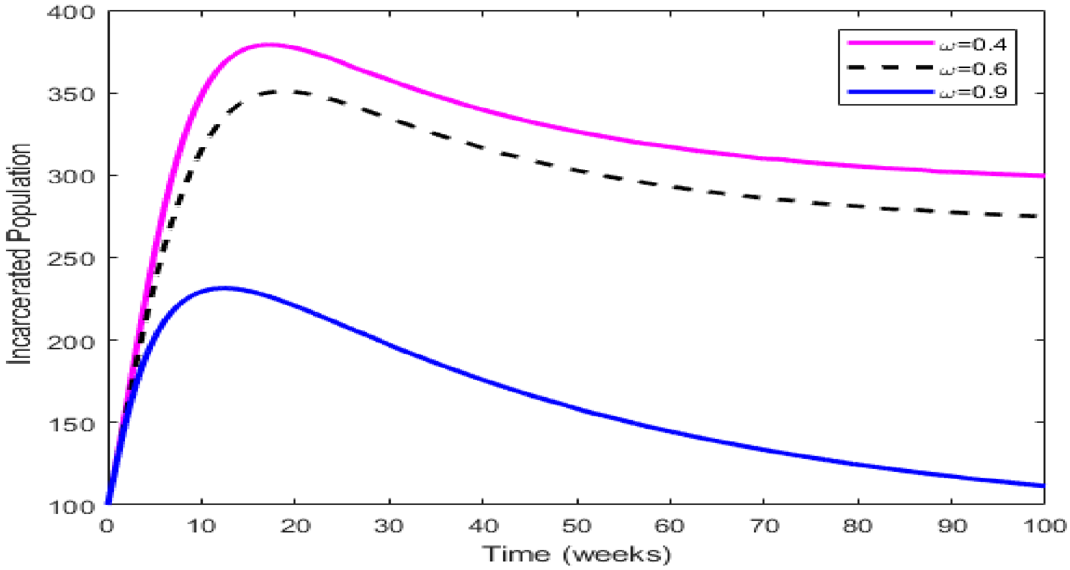

Figure 4.4: Simulation results showing the number of incarcerated human with the variation of employment rate

The figure 4.4 shows the number of incarcerated human with varying rate of employment. It shows that the higher the rate of employment, the lower the number of incarcerated individuals. When the employment rate is high, the number of burglars will significantly reduces,therefore the number of those who will be incarcerated as a results of burglary practices decline. When employment rate is 04, unemployment is 0.6. The graph initially rises highly upto t=12, after the peak, it declines and stabilizes at a higher level. This reflect that the number of incarcerated human remains high compered to when employment rate is 0.6 or 0.9. At $\omega = 0.9$, the number of incarcerated human decline significantly and later tends a smaller value. This means that after a long period of time, the community will be experiencing low burglary incidences when employment rate is high since incarcerated individuals are minimal.

CHAPTER FIVE

DISCUSSION CONCLUSION AND RECOMMENDATIONS

5.1 Introduction

In this chapter, the discussion, conclusion and recommendations of this study are presented. The recommendations are in line with the results of the study as in Chapter 3 and 4.

5.2 Discussion and Conclusions

This study focused on developing a model of dynamics of burglary incorporating unemployment in Kenya. The main focus was to investigate the impact of unemployment on the spread of burglary incidences. From the analytical analysis the effective reproduction number R_e is shown to be directly affected by parameters $\delta, \tau, \alpha, \rho, \gamma, \lambda, \mu$ and ω. The β measures the force of influence of transmission of individuals from susceptible to exposed class and is defined by other parameters as indicated in chapter 3. The analysis shows that the higher the rate of δ, τ, and λ the greater the force of influence, and subsequently, the higher the prevalence of burglary in the society since effective reproduction number will be greater that one. Employment rate ω in inversely proportional to effective reproduction number R_e and serves as a control measure of burglary incidences in Kenya. For instance, when $\omega = 0.4, R_e = 3.1724$ and when $\omega = 0.9, R_e = 0.5287$ which is less than one, assuming all other parameters are kept constant. This suggest that an increase in employment rate decreases R_e, thereby decreases burglary incidences in the society. These results indicate a need to create more job opportunities to decrease the unemployment rate $(1 - \omega)$.

The effective reproduction number was obtained and the analysis indicated that when $R_e < 1$, the burglary free equilibrium is both locally and globally asymptotically stable. However, due to the nature and dynamics of burglary, it is not very easy to eradicate it completely but can be alleviated to a manageable level that does not pose a threat to the development of an economy when $R_e < 1$. Conversely, when $R_e > 1$, the endemic state of burglary is both locally and globally asymptotically stable. This means that burglary will persist in society, remaining a threat unless proper interventions are implemented to curb the unemployment rate $(1-\omega)$.

Sensitivity analysis of the model parameters also revealed that R_e is highly sensitive to the employment rate ω. Increasing ω decreases R_e, thereby reducing burglary incidences. Simulation analysis further demonstrated that a low employment rate contributes to burglary incidences by increasing the force of influence β which in turn increases the transmission rate from the susceptible to the exposed class. With the introduction of incarceration, the rate of burglary decreases as the incarceration rate increases. Employment opportunities reform society by reducing the effect of interactions between susceptible individuals and burglars, thereby increasing susceptibility. Employment also brings positive changes to individuals who have gone through the legal justice process, thus reducing recidivism.

5.3 Recommendations and Future work

Burglary is one of the crimes that has been too costly and challenging for the Kenyan government to address, as it requires significant resources to maintain prison expenses, judicial systems, and establish more police stations in areas prone to burglary.

This study recommends the implementation of employment creation programs and economic empowerment initiatives to mitigate burglary cases. Such measures could help reduce crime rates and improve societal stability. Additionally, the study suggests that law enforcement and rehabilitation strategies should be evaluated within the context of burglary to assess their effectiveness in crime reduction. Policymakers should also determine the optimal timing for implementing these strategies to maximize their impact on reducing burglary cases. For future work, other researchers should consider the following:

1. This study can be extended to incorporate other interventions, such as vocational training, control strategies, government inclusivity, and guidance and counseling programs, among others.
2. Optimal control analysis may also be considered for designing cost-effective interventions that achieve the desired impact at minimal cost.

While research by [10] establishes unemployment as a primary driver of criminal activity, its model examines crime as a generalized phenomenon and crucially omits the role of **recidivism** —the relapse of individuals back into criminal behavior after incarceration. Furthermore, it treats criminal influence as a static parameter, failing to dynamically link it to the prevailing employment rate. This broad approach fails to capture the feedback loop where released offenders, often facing severe employment barriers, re-enter criminal activities, thus perpetuating the cycle of crime. Consequently, a significant gap exists in quantitatively modeling how the employment rate modulates the force of criminal influence and how this interaction, combined with recidivism, propels specific offenses like burglary within unique socioeconomic contexts like Kenya. This study addresses that gap by developing a targeted mathematical model focused exclusively on burglary patterns. Our model innovates by incorporating a **released compartment (R)** with a recidivism parameter (θ) and, most importantly, by defining the force of influence β

as a function of the employment rate (ω), specifically $\beta = \frac{\delta\tau(1-\omega)(B+\lambda E)}{N}$. This formulation explicitly captures how increases in employment dampen the transmission of criminal behavior. By moving from a general crime model to a crime-specific one that integrates both recidivism and an employment-dependent force of influence, we can identify precise, dynamic intervention points, providing policymakers with strategies that simultaneously target economic incentives and rehabilitative pathways.

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